Autumn
Scheme of learning
Year 5

#MathsEveryoneCan

White Rose Maths

The **White Rose Maths** schemes of learning

Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

Putting number first

Our schemes have number at their heart.
A significant amount of time is spent reinforcing number in order to build competency and ensure children can confidently access the rest of the curriculum.

Depth before breadth

Our easy-to-follow schemes support teachers to stay within the required key stage so that children acquire depth of knowledge in each topic. Opportunities to revisit previously learned skills are built into later blocks.

Working together

Children can progress through the schemes as a whole group, encouraging students of all abilities to support each other in their learning.

Fluency, reasoning and problem solving

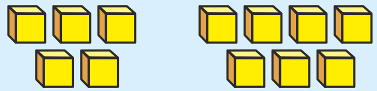
Our schemes develop all three key areas of the National Curriculum, giving children the knowledge and skills they need to become confident mathematicians.

Concrete – Pictorial – Abstract (CPA)

Research shows that all children, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

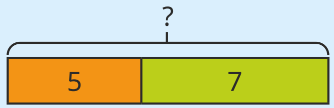
Concrete

Children should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



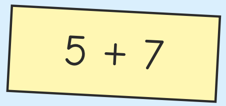
Pictorial

Alongside concrete resources, children should work with pictorial representations, making links to the concrete.
Visualising a problem in this way can help children to reason and to solve problems.



Abstract

With the support of both the concrete and pictorial representations, children can develop their understanding of abstract methods.

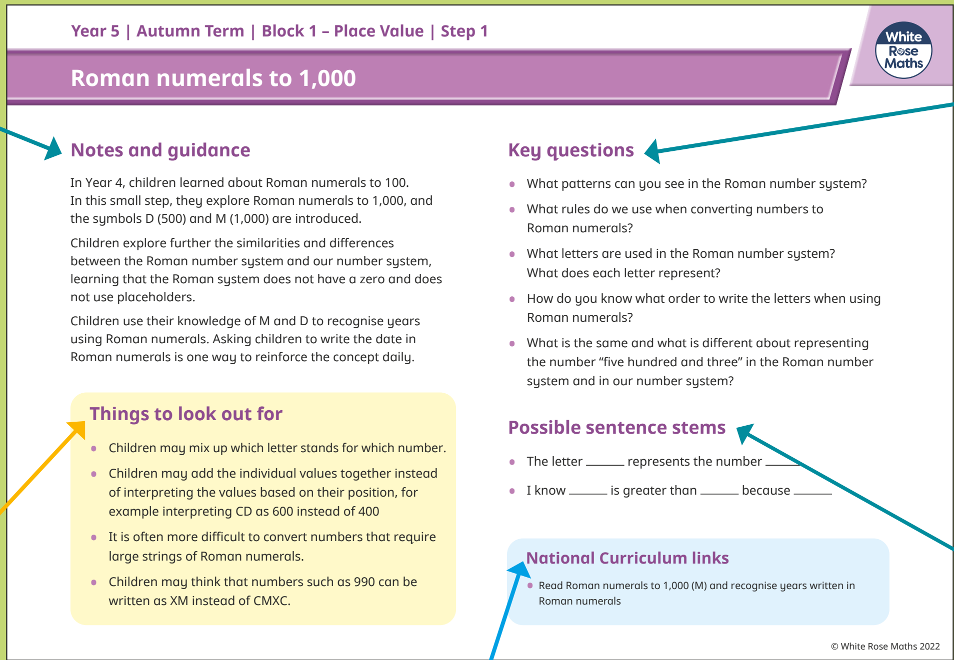


If you have questions about this approach and would like to consider appropriate CPD, please visit www.whiterosemaths.com to find a course that's right for you.

Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, and we provide comprehensive teacher guidance for each one. Here are the features included in each step.

Notes and guidance that provide an overview of the content of the step and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.



Year 5 | Autumn Term | Block 1 – Place Value | Step 1

Roman numerals to 1,000

Notes and guidance

In Year 4, children learned about Roman numerals to 100. In this small step, they explore Roman numerals to 1,000, and the symbols D (500) and M (1,000) are introduced.

Children explore further the similarities and differences between the Roman number system and our number system, learning that the Roman system does not have a zero and does not use placeholders.

Children use their knowledge of M and D to recognise years using Roman numerals. Asking children to write the date in Roman numerals is one way to reinforce the concept daily.

Things to look out for

- Children may mix up which letter stands for which number.
- Children may add the individual values together instead of interpreting the values based on their position, for example interpreting CD as 600 instead of 400
- It is often more difficult to convert numbers that require large strings of Roman numerals.
- Children may think that numbers such as 990 can be written as XM instead of CMXC.

Key questions

- What patterns can you see in the Roman number system?
- What rules do we use when converting numbers to Roman numerals?
- What letters are used in the Roman number system? What does each letter represent?
- How do you know what order to write the letters when using Roman numerals?
- What is the same and what is different about representing the number "five hundred and three" in the Roman number system and in our number system?

Possible sentence stems

- The letter ___ represents the number ___
- I know ___ is greater than ___ because ___

National Curriculum links

- Read Roman numerals to 1,000 (M) and recognise years written in Roman numerals

Key questions that can be posed to children to develop their mathematical vocabulary and reasoning skills, digging deeper into the content.

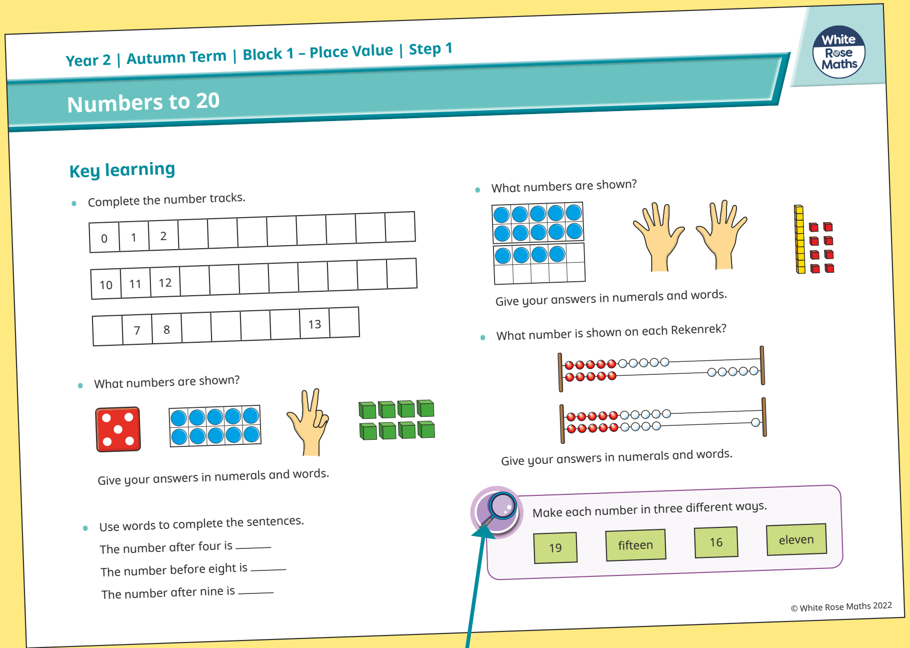
Things to look out for, which highlights common mistakes, misconceptions and areas that may require additional support.

National Curriculum links to indicate the objective(s) being addressed by the step.

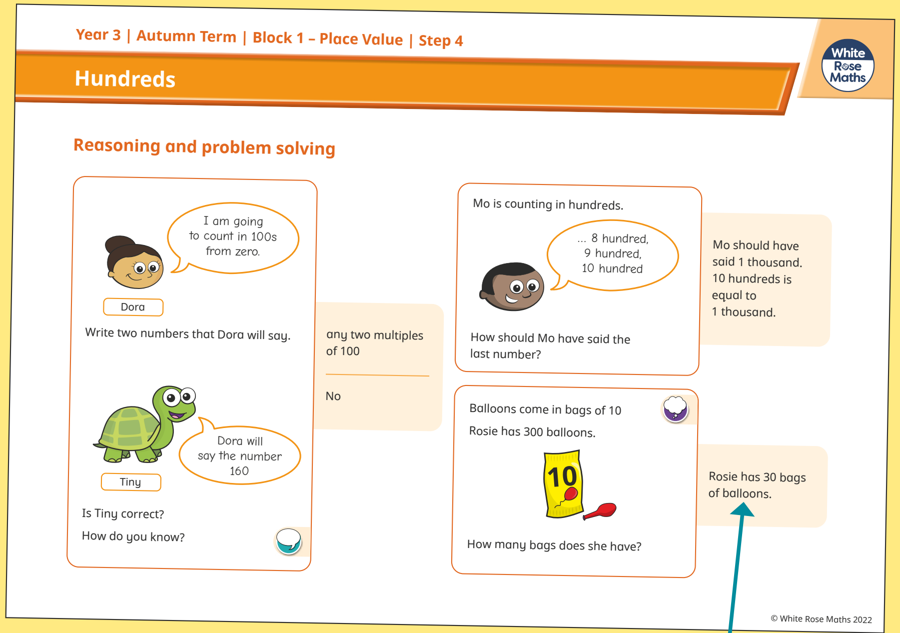
Possible sentence stems to further support children's mathematical language and to develop their reasoning skills.

Teacher guidance

A **Key learning** section, which provides plenty of exemplar questions that can be used when teaching the topic.



Reasoning and problem-solving activities and questions that can be used in class to provide further challenge and to encourage deeper understanding of each topic.



Activity symbols that indicate an idea can be explored practically

Answers provided where appropriate

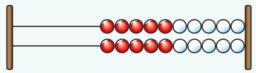
Activities and symbols

Key Stage 1 activities

Key Stage 1 includes more hands-on activities alongside questions.

An activity to be led by the teacher

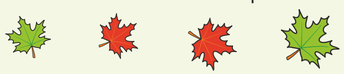
Use a Rekenrek in the ready position.



Ask children to show a number on their Rekenrek.

An outside activity or one that uses resources from nature

Find some seeds and leaves to represent Autumn.



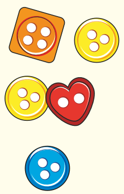
Ask children to sort the objects in three different ways and then compare their answers with a partner.

An activity introduced by a reading from an appropriate fiction or non-fiction book

Read *The Button Box* by M Reid.

Give children a selection of buttons and ask them to sort the buttons in as many different ways as they can.

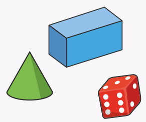
Encourage them to think about size, shape, colour and number of holes.



An investigation

Give children a selection of 3D shapes.

Ask children to sort the objects into two groups and then challenge a partner to say how the objects have been sorted.



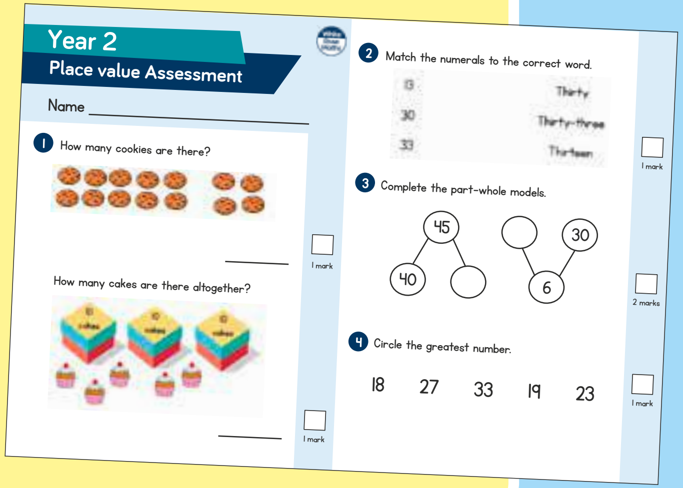
Key Stage 1 and 2 symbols

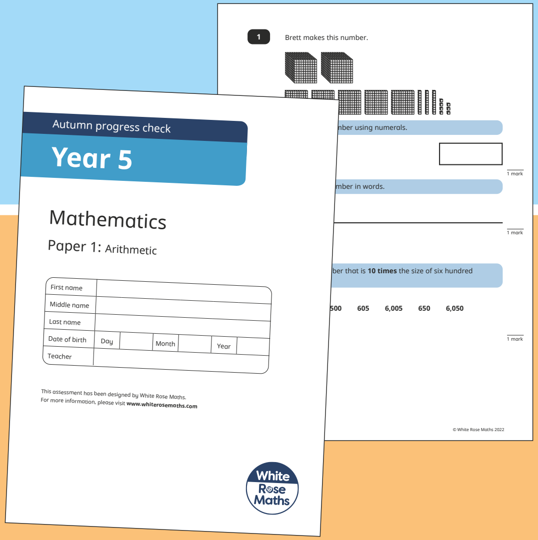
The following symbols are used to indicate:

- concrete resources might be useful to help answer the question
- a bar model might be useful to help answer the question
- drawing a picture might help children to answer the question
- children talk about and compare their answers and reasoning
- a question that should really make children think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.

Free supporting materials

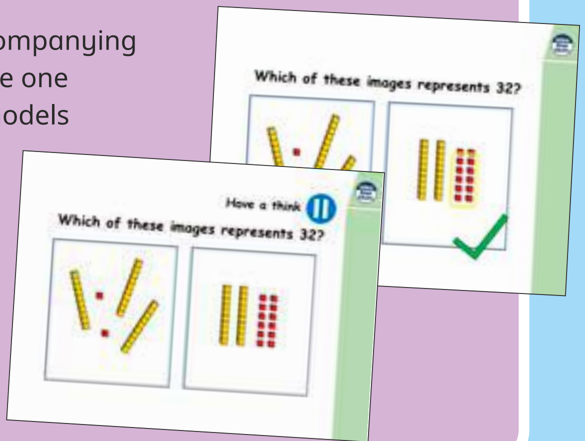
End-of-block assessments to check progress and identify gaps in knowledge and understanding.



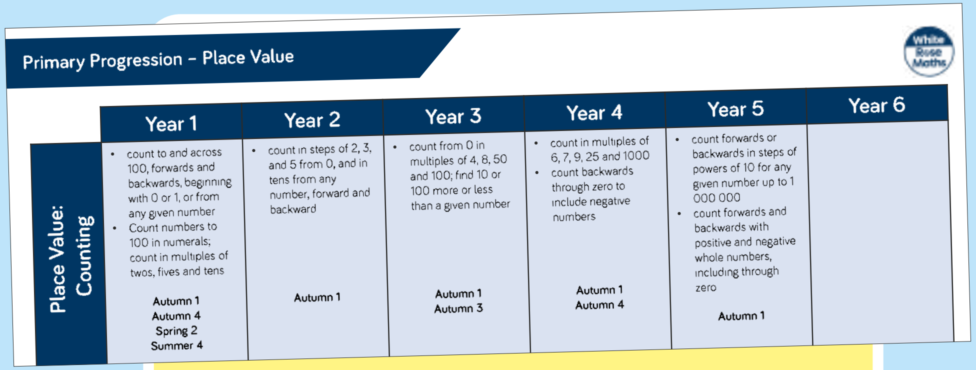


End-of-term assessments for a more summative view of where children are succeeding and where they may need more support.

Each small step has an accompanying **home learning video** where one of our team of specialists models the learning in the step. These can also be used to support students who are absent or who need to catch up content from earlier blocks or years.



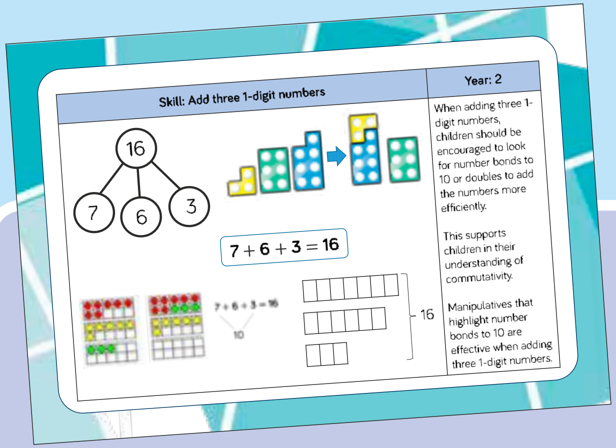
Free supporting materials



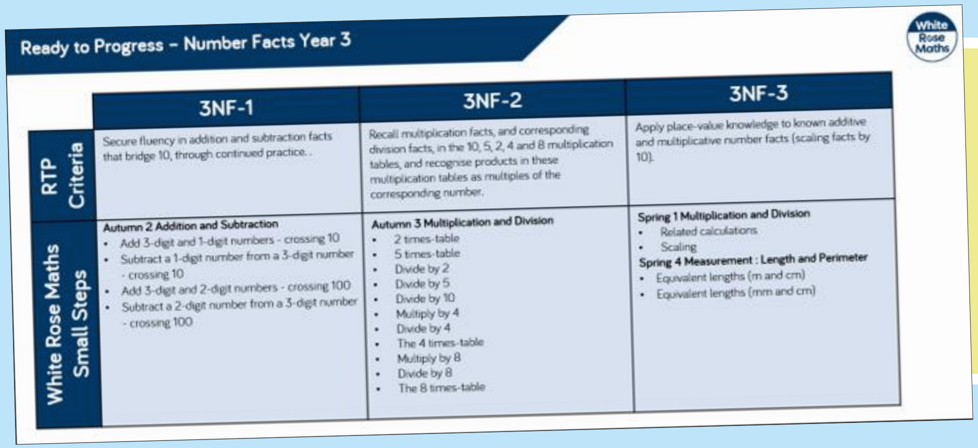
Primary Progression – Place Value

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Place Value: Counting	• count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number • Count numbers to 100 in numerals; count in multiples of twos, fives and tens Autumn 1 Autumn 4 Spring 2 Summer 4	• count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward Autumn 1	• count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number Autumn 1 Autumn 3	• count in multiples of 6, 7, 9, 25 and 1000 • count backwards through zero to include negative numbers Autumn 1 Autumn 4	• count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 • count forwards and backwards with positive and negative whole numbers, including through zero Autumn 1	

National Curriculum progression to indicate how the schemes of learning fit into the wider picture and how learning progresses within and between year groups.



Calculation policies that show how key approaches develop from Year 1 to Year 6.



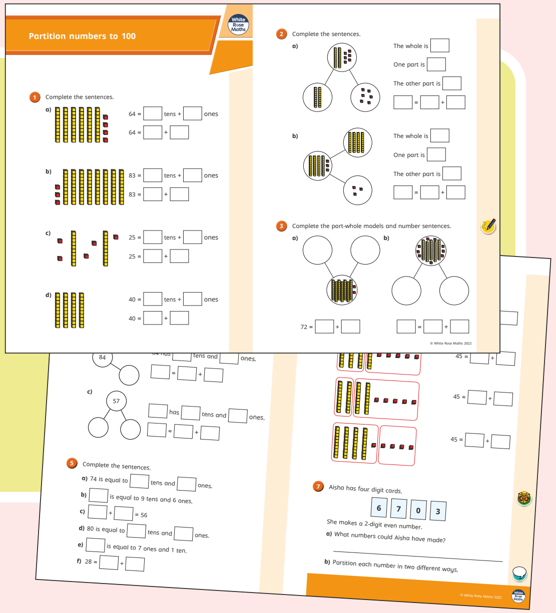
Ready to Progress – Number Facts Year 3

	3NF-1	3NF-2	3NF-3
RTP Criteria	Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10).
White Rose Maths Small Steps	**Autumn 2 Addition and Subtraction** • Add 3-digit and 1-digit numbers - crossing 10 • Subtract a 1-digit number from a 3-digit number - crossing 10 • Add 3-digit and 2-digit numbers - crossing 100 • Subtract a 2-digit number from a 3-digit number - crossing 100	**Autumn 3 Multiplication and Division** • 2 times-table • 5 times-table • Divide by 2 • Divide by 5 • Divide by 10 • Multiply by 4 • Divide by 4 • The 4 times-table • Multiply by 8 • Divide by 8 • The 8 times-table	**Spring 1 Multiplication and Division** • Related calculations • Scaling **Spring 4 Measurement : Length and Perimeter** • Equivalent lengths (m and cm) • Equivalent lengths (mm and cm)

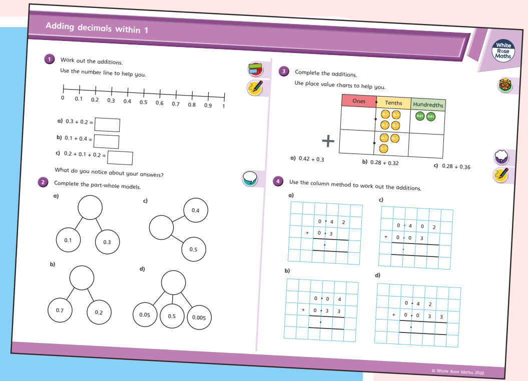
Ready to progress mapping that shows how the schemes of learning link to curriculum prioritisation.

Premium supporting materials

Worksheets to accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.

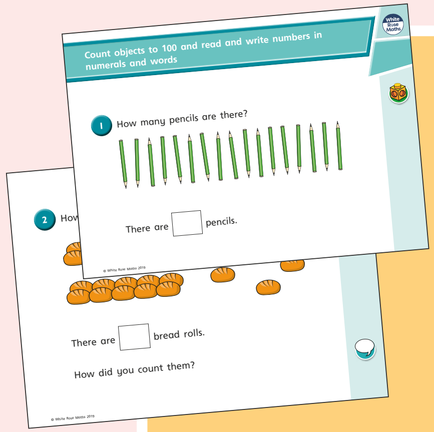


Display versions of the worksheet questions for front of class/whole class teaching.

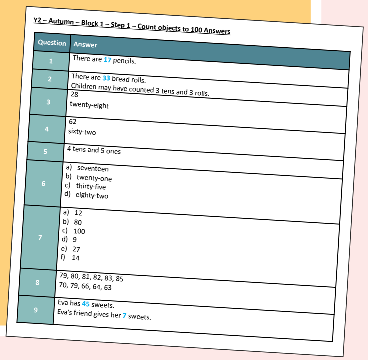


Also available as printed **workbooks**, per block.

PowerPoint™ versions of the worksheet questions to incorporate them into lesson planning.

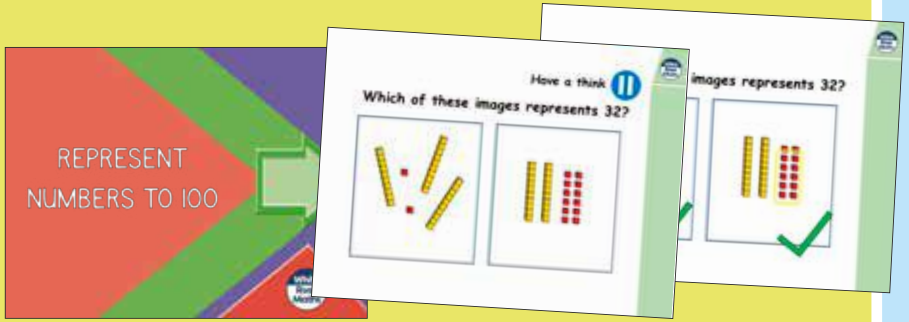


Answers to all the worksheet questions.

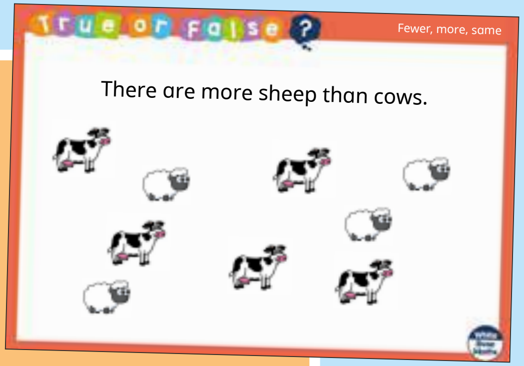


Premium supporting materials

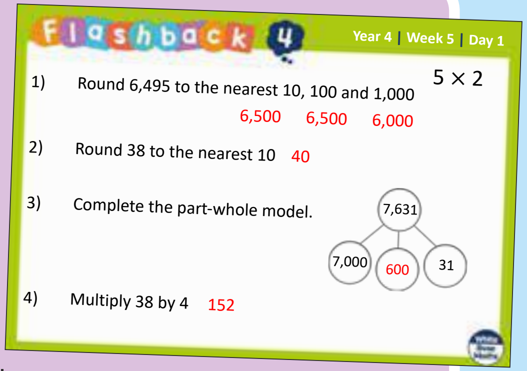
Teaching slides that mirror the content of our home learning videos for each step. These are fully animated and editable, so can be adapted to the needs of any class.



A **true or false** question for every small step in the scheme of learning. These can be used to support new learning or as another tool for revisiting knowledge at a later date.



Flashback 4 starter activities to improve retention.
Q1 is from the last lesson;
Q2 is from last week;
Q3 is from 2 to 3 weeks ago;
Q4 is from last term/year.
There is also a bonus question on each one to recap topics such as telling the time, times-tables and Roman numerals.



Topic-based CPD videos
As part of our on-demand CPD package, our maths specialists provide helpful hints and guidance on teaching topics for every block in our schemes of learning.

Meet the characters

Our class of characters bring the schemes to life, and will be sure to engage learners of all ages and abilities. Follow the children and their class pet, Tiny the tortoise, as they explore new mathematical concepts and ideas.

Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number **Place value**			Number **Addition and subtraction**		Number **Multiplication and division A**			Number **Fractions A**			
Spring	Number **Multiplication and division B**			Number **Fractions B**		Number **Decimals and percentages**			Measurement **Perimeter and area**		**Statistics**	
Summer	Geometry **Shape**			Geometry **Position and direction**		Number **Decimals**			Number **Negative numbers**	Measurement **Converting units**		Measurement **Volume**

Autumn Block 1

Place value

Small steps

Step 1	Roman numerals to 1,000
Step 2	Numbers to 10,000
Step 3	Numbers to 100,000
Step 4	Numbers to 1,000,000
Step 5	Read and write numbers to 1,000,000
Step 6	Powers of 10
Step 7	10/100/1,000/10,000/100,000 more or less
Step 8	Partition numbers to 1,000,000

Small steps

Step 9	Number line to 1,000,000
Step 10	Compare and order numbers to 100,000
Step 11	Compare and order numbers to 1,000,000
Step 12	Round to the nearest 10, 100 or 1,000
Step 13	Round within 100,000
Step 14	Round within 1,000,000

Roman numerals to 1,000

Notes and guidance

In Year 4, children learned about Roman numerals to 100. In this small step, they explore Roman numerals to 1,000, and the symbols D (500) and M (1,000) are introduced.

Children explore further the similarities and differences between the Roman number system and our number system, learning that the Roman system does not have a zero and does not use placeholders.

Children use their knowledge of M and D to recognise years using Roman numerals. Asking children to write the date in Roman numerals is one way to reinforce the concept daily.

Things to look out for

- Children may mix up which letter stands for which number.
- Children may add the individual values together instead of interpreting the values based on their position, for example interpreting CD as 600 instead of 400
- It is often more difficult to convert numbers that require large strings of Roman numerals.
- Children may think that numbers such as 990 can be written as XM instead of CMXC.

Key questions

- What patterns can you see in the Roman number system?
- What rules do we use when converting numbers to Roman numerals?
- What letters are used in the Roman number system? What does each letter represent?
- How do you know what order to write the letters when using Roman numerals?
- What is the same and what is different about representing the number "five hundred and three" in the Roman number system and in our number system?

Possible sentence stems

- The letter ______ represents the number ______
- I know ______ is greater than ______ because …

National Curriculum links

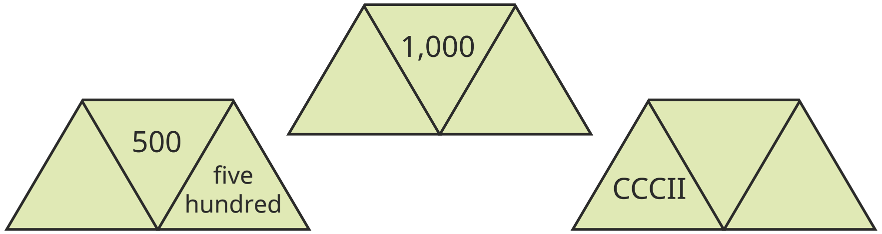
- Read Roman numerals to 1,000 (M) and recognise years written in Roman numerals

Roman numerals to 1,000

Key learning

- Each diagram should show a number in Roman numerals, digits and words.

  Complete the diagrams.



- Match the Roman numerals to the numbers.

DC	460
CD	950
CCCXX	400
DXC	590
CML	600
CDLX	320

- Here is a date written in Roman numerals.

  XXI / IX / MMXV

  What day of the month is shown?

  What month is shown?

  What year is shown?

- Here are the end credits of two films.

  The Roman numerals show the year the films were made.

In what year was the older film made?

In what year was the more recent film made?

How long was there between the making of the two films?

Give your answer in Roman numerals.

Roman numerals to 1,000

Reasoning and problem solving

Work out CCCL + CL.

Give your answer in Roman numerals.

Write five calculations, using Roman numerals, that give the same answer.

Compare answers with a partner.

D

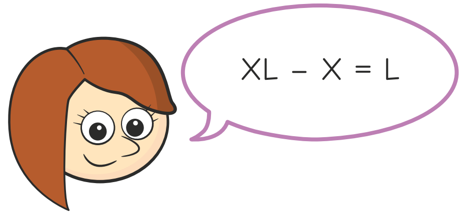
multiple possible answers, e.g.

CD + C

M ÷ II

C + CC + CC

C × V



Do you agree with Rosie?

Explain your answer.

No

Is the statement true or false?

> In Roman numerals,
> 400 is CD,
> so 800 is CDCD.

False

The numbers in the sequence are increasing by CXX each time.

☐, ☐, DCCXC, ☐, ☐

Work out the missing numbers in the sequence.

DL, DCLXX,

CMX, MXXX

Numbers to 10,000

Notes and guidance

Children encountered numbers up to 10,000 in Year 4. In this small step, they revise this learning in preparation for looking at numbers to 100,000 and then 1,000,000

A variety of pictorial and concrete representations are used, including base 10, place value counters, place value charts and part-whole models. In particular, the ability to use place value charts needs to be secure, as this is the main representation used in the coming steps where children learn about 5- and 6-digit numbers.

Children should also be able to add and subtract 10, 100 and 1,000 to and from a given number, using their place value knowledge rather than formal written methods.

Things to look out for

- Children may not yet have fully grasped placeholders, for example reading 208 as twenty-eight.
- Children may rely on the column method of addition and subtraction when this is not necessary.
- Children may not use, or may misplace, the comma when writing numbers greater than or equal to 1,000

Key questions

- What is the value of each digit in the number?
- How can you represent the number in a different way?
- Which digit or digits would change in value if you added a 10/100/1,000 counter?
- How do you write the number in words?

Possible sentence stems

- The value of the ______ in ______ is ______
- The column before/after the ______ column is the ______ column.
- 10 ______ can be exchanged for 1 ______
- 1 ______ can be exchanged for ______

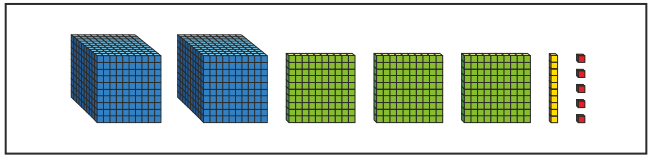
National Curriculum links

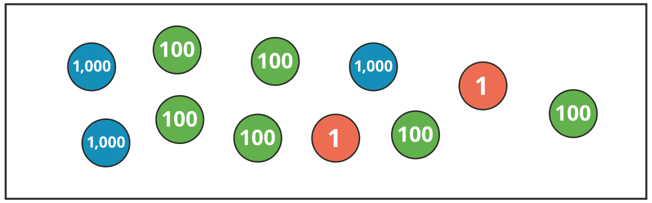
- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000

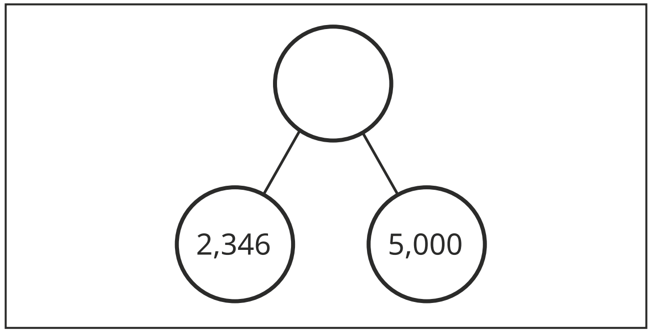
Numbers to 10,000

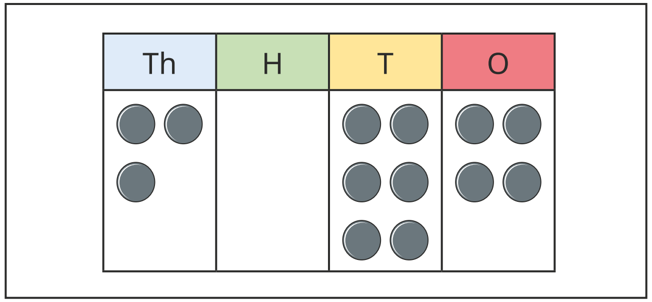
Key learning

- What numbers are shown?

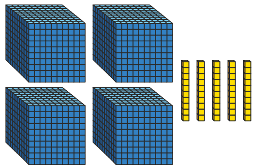


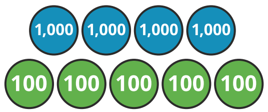


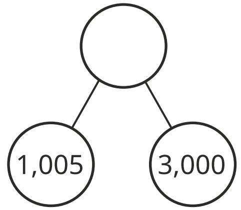




- Match the representations to the numbers.

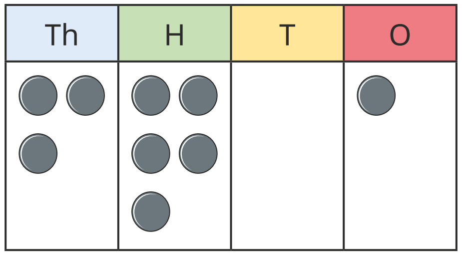






4,005 4,500 4,050

- Show the number 2,536 in three different ways.
- What number is shown in the place value chart?



What will the number be if you add a counter to the thousands column?

What will the number be if you take two counters away from the hundreds column?

Numbers to 10,000

Reasoning and problem solving

Filip has made five numbers using the digits 1, 2, 3 and 4

He is using a letter to represent each digit.

Here are his numbers.

AABCD
ACDCB
DCABA
CDADC
BDAAB

Use the clues to work out each number.

- The first number in the list is the greatest number.
- The digits in the fourth number add up to 12
- The third number is the smallest number.

44,231
43,132
13,424
31,413
21,442

Work out the missing numbers.

	Add 10	Add 100	Add 1,000
7,516			
			5,209
		6,025	
	3,001		

	Add 10	Add 100	Add 1,000
7,516	7,526	7,616	8,516
4,209	4,219	4,309	5,209
5,925	5,935	6,025	6,925
2,991	3,001	3,091	3,991

Numbers to 100,000

Notes and guidance

In this small step, children build on the Year 4 learning revised in the previous step, and explore numbers up to 100,000

They are introduced to the ten-thousands column in a place value chart and begin to understand the multiples of 10,000. This can be reinforced using a number line to 100,000

Both place value counters and plain counters are used in place value charts, allowing for discussion about the values of the columns.

Children estimate the position of numbers such as 65,048 on a number line, preparing them for rounding later in this block.

Things to look out for

- Children are likely to use "thousands" and "millions" in everyday speech more often than "tens of thousands" or "hundreds of thousands", so they may miss out place value columns in between.
- Children may find numbers with several placeholders difficult, for example 40,020
- Children may need support in deciding when to use the word "and" when saying numbers, for example 3,100 does not use "and" but 3,010 does.

Key questions

- Counting in 1,000s, what would you say after "nine thousand"?
- Counting in 10,000s, what would you say after "sixty thousand"?
- How can you represent the number 65,000 using a number line?
- What is the value of each digit in the number?
- If 100,000 is the whole, what could the parts be?

Possible sentence stems

- The value of the ______ in ______ is ______
- The column before/after the ______ column is the ______ column.

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000

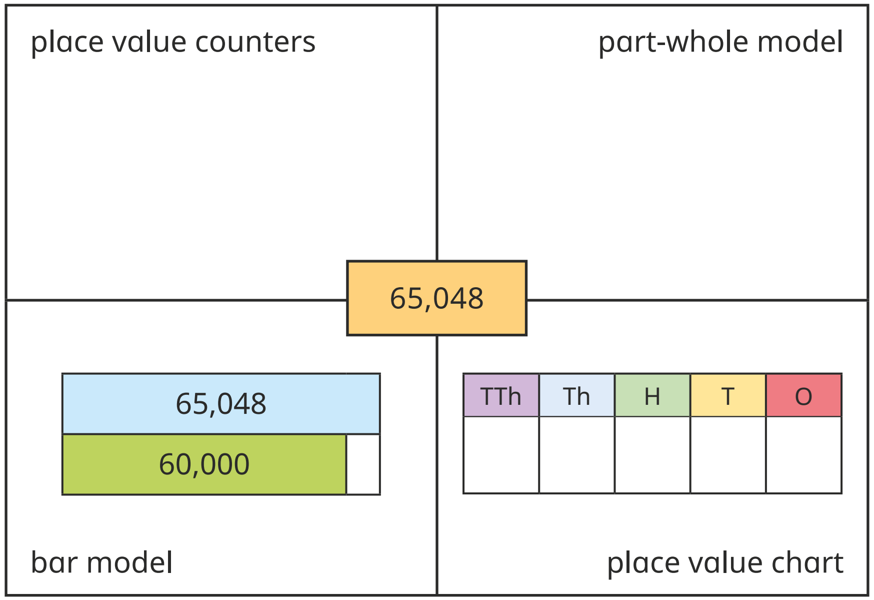
Numbers to 100,000

Key learning

- What number is shown on the place value chart?

TTh	Th	H	T	O
10,000 10,000 10,000 10,000	1,000	100 100 100	10 10	1 1

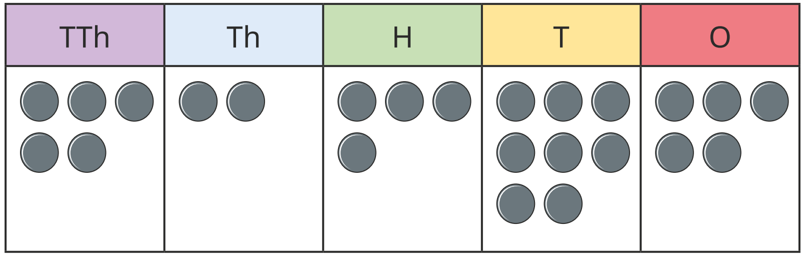
- Complete the grid to show the number in different ways.



- Find the missing numbers.
  - 59,000 = 50,000 + ______
  - ______ = 30,000 + 1,700 + 80
  - 75,480 = ______ + 3,000 + ______

  Do any of the questions have more than one possible answer?

- A number is shown in the place value chart.



What number is represented?

A counter is removed from the thousands column.
What number is represented now?

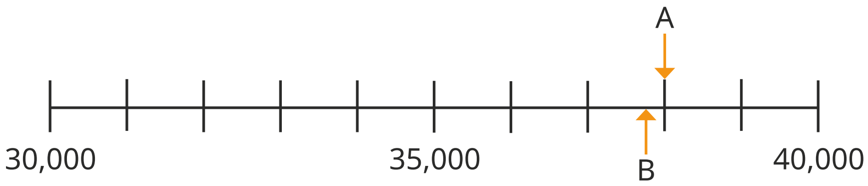
A counter is then added to the tens column.
What number is represented now?

- Count down in 10,000s from 157,000 to 27,000

Numbers to 100,000

Reasoning and problem solving

Here is a number line.



What is the value of A?

B is 100 less than A.
What is the value of B?

C is 1,000 less than B.
Label C on the number line.

A = 38,000 B = 37,900

Write as many different numbers as you can, using each word no more than once.

You do not need to use all the words each time.

| and | four | thousand |
| one | hundred | |

1, 4, 100, 104, 400, 401, 1,000, 1,004, 1,400, 4,000, 4,001, 4,100, 100,000, 100,004, 104,000, 400,000, 400,001, 401,000

List all the 5-digit numbers you can make using the digit cards.

1 **2** **0** **0** **0**

12,000, 10,200, 10,020, 10,002, 21,000, 20,100, 20,010, 20,001

Numbers to 1,000,000

Notes and guidance

In this small step, children build on the previous steps and explore numbers up to 1,000,000

Children learn that the pattern for thousands in a place value chart follows the same pattern as that of the ones: ones, tens, hundreds, (one) thousands, ten thousands, hundred thousands. Children recognise large numbers presented in a variety of ways using familiar models. Reading numbers is touched on in this step and then developed in the next step, which also looks at writing numbers in words.

Partitioning is introduced but will be covered in more detail later in the block.

Things to look out for

- Children may find it difficult to conceptualise such large numbers as they lie outside their everyday experience and cannot easily be represented concretely.
- Unless they are confident with the previous step, children may think that place value columns go in the order ones, tens, hundreds, thousands, millions.
- Children may find numbers with several placeholders difficult.

Key questions

- Where do the commas go when writing one million in numerals?
- How does a place value chart help you to represent large numbers?
- What is the value of each digit in this number?
- Are 6-digit numbers always greater in value than 5-digit numbers?
- When do you use placeholders in numbers?
- If one million is the whole, what could the parts be?

Possible sentence stems

- The value of the ______ in ______ is ______
- The column before/after the ______ column is the ______ column.

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000

Numbers to 1,000,000

Key learning

- What number is shown in each place value chart?

  Give your answers in numerals.

HTh	TTh	Th	H	T	O
●●●●	●●	●●	●●●●	●●●●	●

Thousands			Ones		
H	T	O	H	T	O
●●●●	●●	●●	●●●●	●●●●	●

  What is the same and what is different about these place value charts?

- Use counters to make the numbers on a place value chart.

  32,651 463,215 320,154 60,020

- Count in 100,000s from zero to 1 million.

- Use counters to make the numbers on the place value chart.

  372,524 206,401 300,042 71,560

Thousands			Ones		
H	T	O	H	T	O

  How would you say the numbers?

- What is the value of the 4 in each number?

  124,306 245,812 402,001

  321,247 604,513 45,872

- Write four numbers that have a 3 in the hundreds column.

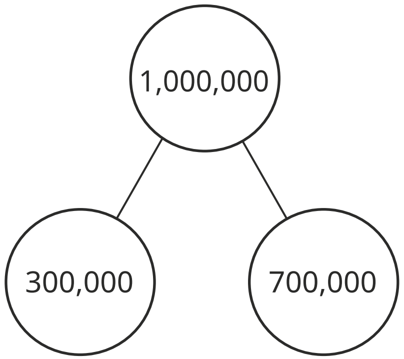
  Each number should have a different number of digits.

Numbers to 1,000,000

Reasoning and problem solving

Here are two ways of partitioning one million into multiples of 100,000

1,000,000	
600,000	400,000



How many other ways can you find to partition one million into multiples of 100,000?

Show your answers as bar models and part-whole models.

There are four more ways:

- 0 and 1,000,000
- 100,000 and 900,000
- 200,000 and 800,000
- 500,000 and 500,000

The numbers can be written in either order.

Use the digit cards to make as many 6-digit numbers as you can.

5	5	5
0	0	0

What is the greatest number you can make?

What is the smallest number you can make?

What is the difference between the greatest and smallest numbers?

Ten 6-digit numbers can be made:

555,000	505,050
550,500	505,005
550,050	500,550
550,005	500,505
505,500	500,055

555,000

500,055

54,945

Read and write numbers to 1,000,000

Notes and guidance

Children should be secure with the place value of numbers to 1,000,000. In this small step, they develop their skill at reading and writing large numbers in words, which has been touched on in earlier steps.

While the spelling of the individual words is important, the focus of the step is the structure of the written words, for example we read and write 4,100 as "four thousand one hundred" but 4,010 as "four thousand and ten".

Using a comma as a separator helps with reading and writing numbers in two parts, and a part-whole model or place value chart can be used to support this.

Things to look out for

- Children who find the "teen" numbers difficult may have problems with numbers such as 317,413
- Children may find reading and writing numbers with placeholders (for example, 700,011) difficult.
- Knowing when to use the word "and" within a number can sometimes cause confusion.

Key questions

- When a number is written with commas, what do the numbers before/after each comma represent?
- How can this number be represented using a part-whole model? What parts would it be sensible to use?
- How do you write "1,000,000" in words?
- When do you use the word "and" when reading or writing a number?

Possible sentence stems

- The number before/after the comma is ______. This part of the number is said/written as ______
- The whole of the number is said/written as ______

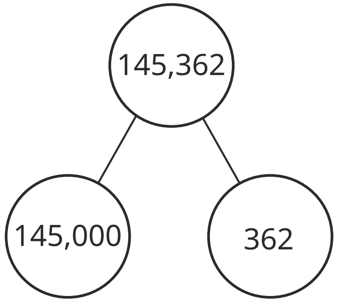
National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Solve number problems and practical problems involving the above

Read and write numbers to 1,000,000

Key learning

- Scott is using a part-whole model to help write the number 145,362 in words.

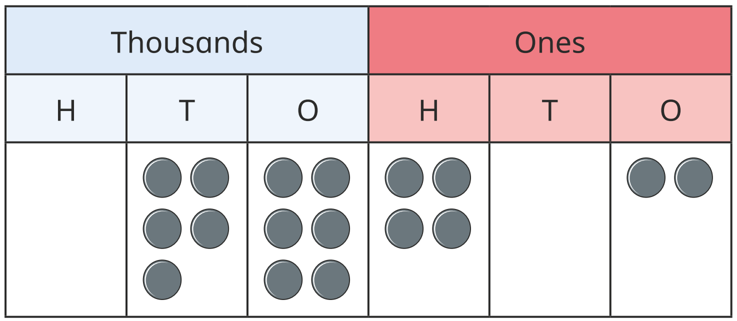


one hundred and forty-five thousand and three hundred and sixty-two

Scott has made one mistake.

Write 145,362 correctly in words.

- 56,402 is shown in the place value chart.



Write the number 56,402 in words.

How does the place value chart help you?

- Write the numbers in words.

1,256 | 12,560 | 125,600 | 120,560 | 120,506

You could write the numbers in a place value chart to help you.

- A number is made up of 2 ten-thousands, 5 hundreds and 7 ones.

Show the number on a place value chart.

Write the number in words and numerals.

- Write the numbers in numerals.

three hundred and six thousand and fifteen

three hundred and six thousand and fifty

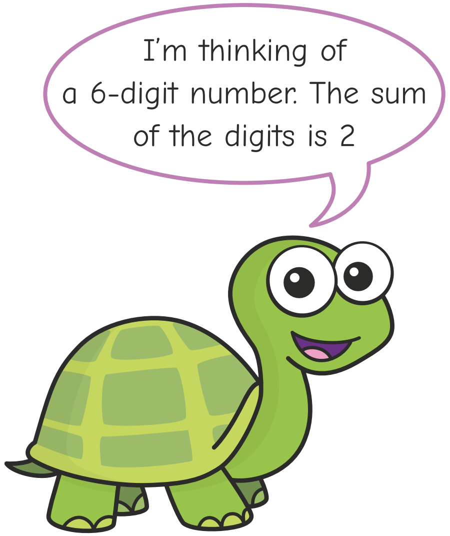
three hundred and fifteen thousand and six

- Use place value counters to make the number "half a million".

Write the number "half a million" in numerals.

Read and write numbers to 1,000,000

Reasoning and problem solving



Find all the possible numbers Tiny could be thinking of.

Give your answers in words and numerals.

Investigate with different digit sums.

What do you notice?

200,000
two hundred thousand

110,000
one hundred and ten thousand

101,000
one hundred and one thousand

100,100
one hundred thousand, one hundred

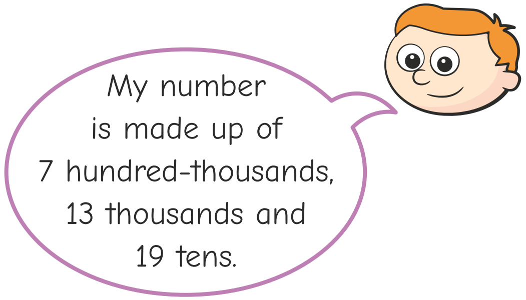
100,010
one hundred thousand and ten

100,001
one hundred thousand and one

When written in words, what is the first number that includes the letter "a"?

one hundred and one

Ron is thinking of a number.



What is 1,000 less than Ron's number?

What is 10 more than Ron's number?

Give your answers in words.

seven hundred and twelve thousand, one hundred and ninety

seven hundred and thirteen thousand, two hundred

Powers of 10

Notes and guidance

In this small step, children further develop their understanding of place value by exploring the relationship between numbers in different columns.

As well as adjacent columns, they look at columns that are further apart, for example considering the number of tens needed to make 1,000 and then multiples of 1,000. Children use both place value charts and Gattegno charts to support their understanding. You could demonstrate exchanging with place value counters as extra support if needed.

Multiplication by 10, 100 and 1,000 is covered in detail later in the term. The focus here is on the place value of the digits rather than performing calculations.

Things to look out for

- Children may not realise that the overall effect of, for example, × 10 followed by × 10 is × 100
- Children may find it confusing that numbers increase by a factor of 10 horizontally on a place value chart but vertically on a Gattegno chart.

Key questions

- How can you tell if a number is a power of 10?
- Is this number a multiple of a power of 10? How can you tell?
- If you move a digit one place to the left in a place value chart, how many times greater is the value of the digit?
- If you move a digit two places to the left in a place value chart, how many times greater is the value of the digit?
- What patterns can you see in the Gattegno chart?

Possible sentence stems

- There are ______ hundreds in 1,000 and ______ thousands in ______. This means there are ______ hundreds in ______
- ______ is ______ the size of ______

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Solve number problems and practical problems that involve the above

Powers of 10

Key learning

- Make the number 425 on a place value chart.

Thousands			Ones		
H	T	O	H	T	O

Now make the number 4,250

What is the same and what is different?

- How many tens are there in 100?

How many tens are there in 200?

How many tens are there in 210?

How many tens are there in 740?

- How many tens are there in 100?

How many tens are there in 1,000?

How many tens are there in 2,000?

How many hundreds are there in 2,000?

- What number is shown on the Gattegno chart?

100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Use the chart to find the number 100 times the size of the number shown.

Use the chart to make the number one-tenth the size of the number shown.

- Complete the sentences.
  - There are 1,000 metres in a kilometre.

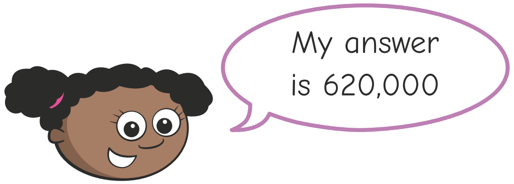
    ________ km is the same distance as 68,000 m.
  - There are 1,000 millimetres in a metre.

    ________ mm is the same length as 803 m.

Powers of 10

Reasoning and problem solving

Whitney and Amir use a Gattegno chart to answer questions.



Whitney

What could Whitney's question be?

Amir

What could Amir's question be?

multiple possible answers, e.g.

What is 10 times the size of 62,000?

multiple possible answers, e.g.

What is 100 times the size of 6,020?

Large areas are measured in hectares.

1 hectare = 10,000 m^2

The area of the Eden Park stadium in New Zealand is 15 hectares.

What is the area of Eden Park in m^2?

How many plots with an area of 100 m^2 could be made in Eden Park?

150,000 m^2

150

1,000 × 1,000 = 1,000,000

How many other calculations using just ones and zeros can you find that have the answer 1,000,000?

multiple possible answers, e.g.

10,000 × 100

100,000 × 10

1,000,000 × 1

100 × 100 × 100

100 × 10 × 1,000

10/100/1,000/10,000/100,000 more or less

Notes and guidance

In this small step, children use place value to find numbers 10/100/1,000/10,000/100,000 more or less than a given number. They need to be able to count both forwards and backwards in steps of powers of 10, and should be encouraged to spot patterns in the sequences formed by doing this. Children could be stretched to consider the rule that connects consecutive terms in the resulting sequences.

As well as finding consecutive values when counting forwards and backwards, children should also be able to find missing numbers that lie between two other given values.

A Gattegno chart is useful to support adding the correct power of 10, and to see what happens when crossing a 10/100/1,000 ... boundary.

Things to look out for

- Children may make errors when they are counting across a multiple of 10, 100, 1,000 ... For example, 2,080, 2,090, 3,000
- More support may be needed when counting backwards.

Key questions

- How can you use a place value chart to find 10/100/1,000 ... more/less than a given number?
- How can you use a Gattegno chart to find 10/100/1,000 ... more/less than a given number?
- How many digits of the number will change if you add 10/100/1,000 ... to the given number?
- What is the same and what is different about the patterns of the numbers vertically and horizontally in a Gattegno chart?

Possible sentence stems

- ______ more/less than ______ is ______
- ______ is ______ more/less than ______

National Curriculum links

- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000
- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit

10/100/1,000/10,000/100,000 more or less

Key learning

- Here is a Gattegno chart showing the number 32,450

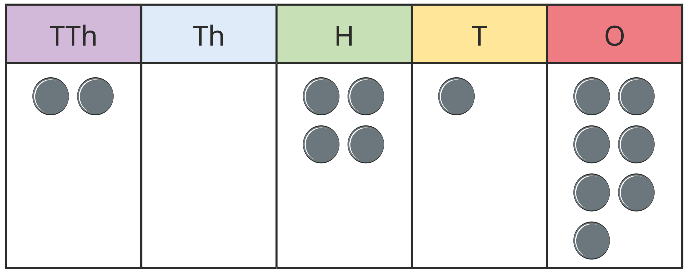
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

What number is 10 more than 32,450?

What number is 100 less than 32,450?

What number is 10,000 less than 32,450?

- 20,417 is shown in the place value chart.



What is 100 more than 20,417?

What is 10 less than 20,417?

What is 1,000 less than 20,417?

- Complete the number tracks.

663	673		693		713	

7,200		7,000			

7,200		6,800			

- Count up in 1,000s starting from 6,240

  Count up in 10,000s starting from 6,240

  Count up in 100,000s starting from 6,240

- Correct the mistake in each number sequence.

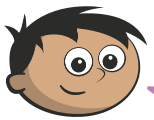
  7,875 , 8,875 , 9,875 , 11,875 , 12,875 , 13,875

  864,664 , 764,664 , 664,664 , 554,664 , 444,664

10/100/1,000/10,000/100,000 more or less

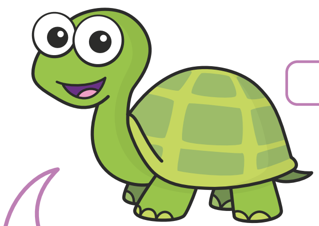
Reasoning and problem solving

Amir is counting in thousands.



3,000, 4,000, 5,000, 6,000, 7,000

Amir



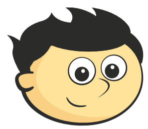
Tiny

The tenth number Amir will say is 14,000 because it is double 7,000

Do you agree with Tiny?

Explain your answer.

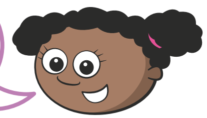
No



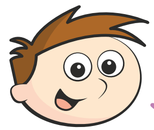
I am counting up in tens from 184
I will include 224

Jack

I am counting up in hundreds from 604
I will include 1,040



Whitney



I am counting up in thousands from 13
I will include 13,000

Teddy

Are the children correct?

Explain how you know.

Jack is correct.

Whitney is incorrect. All her numbers will end in 04

Teddy is incorrect. All his numbers will end in 13

Partition numbers to 1,000,000

Notes and guidance

Children have been partitioning numbers since Year 2. In this small step, they extend their knowledge to deal with larger numbers while consolidating their understanding of the place value columns that have been introduced this year.

They partition numbers in the standard way (for example, into thousands, hundreds, tens and ones) as well as in more flexible ways (for example, 15,875 = 14,875 + 1,000 and 15,875 = 13,475 + 2,400).

Understanding of partitioning, for example changing 62 to 50 + 12, supports methods for addition and subtraction that will be reviewed in the next block.

Things to look out for

- Children may make mistakes with the order of the digits when partitioning/recombining numbers with many digits.
- Children may be less familiar with non-standard partitioning and need the support of, for example, place value counters to see alternatives.
- Children may wish to apply a formal method when the values of the digits in the columns make it more appropriate.

Key questions

- What number is being represented?
- How can place value cards be used to help partition a number?
- If you have 10 hundreds/thousands/ten-thousands, what can these be exchanged for?
- How does knowing that 9 + 5 = 14 help you to work out 9 tens + 5 tens? What about 9 thousands + 5 thousands?
- How else can you say/write "14 tens" or "14 thousands"?

Possible sentence stems

- The value of the first digit is ______
- The value of the next digit is ______
- ______ is equal to ______ thousands, ______ hundreds, ______ tens and ______ ones.

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit

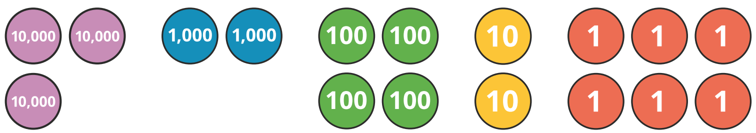
Partition numbers to 1,000,000

Key learning

- Partition the numbers into thousands, hundreds, tens and ones.
  - 6,789 = ______ + ______ + ______ + ______
  - 4,813 = ______ + ______ + ______ + ______

- Complete the number sentences.
  - ______ = 20,000 + 7,000 + 800 + 40 + 3
  - 560,830 = ______ + 60,000 + ______ + 30

- Move the place value counters around and make exchanges to help you complete the partitions.



32,426 = 30,000 + 2,000 + ______ + 20 + 6

32,426 = 20,000 + ______ + 400 + 10 + ______

32,426 = 10,000 + 22,000 + ______ + ______

Is there more than one answer for any of these?

Find other ways to partition the number.

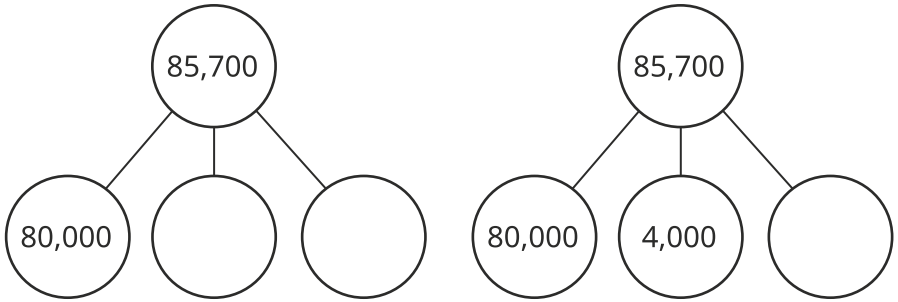
- Aisha is partitioning 45,627

40 + 50,000 + 600 + 2 + 7,000 = 45,627

Explain why Aisha's workings are wrong.

Find the correct total.

- Complete the part-whole models for 85,700



Find three more ways of partitioning 85,700 into three parts.

- Complete the calculations.
  - 367,201 = 200,000 + ______
  - 40,000 + 27,600 + 250 = ______
  - 945,006 = 610,000 + ______ + 6

Partition numbers to 1,000,000

Reasoning and problem solving

Esther is partitioning a number written in Roman numerals.

MMDXL = M + M + D + X + X + X + X

Is Esther correct?

Find some other ways of partitioning the number using Roman numerals.

Esther is correct.

multiple possible answers, e.g.

MM + CD + C + XL

M + D + D + D + XL

Which is the odd one out?

680,000	680 thousands	68 ten-thousands
5 hundred-thousands plus 180 thousands		680 hundreds

680 hundreds is the odd one out, as it is equal to 68,000

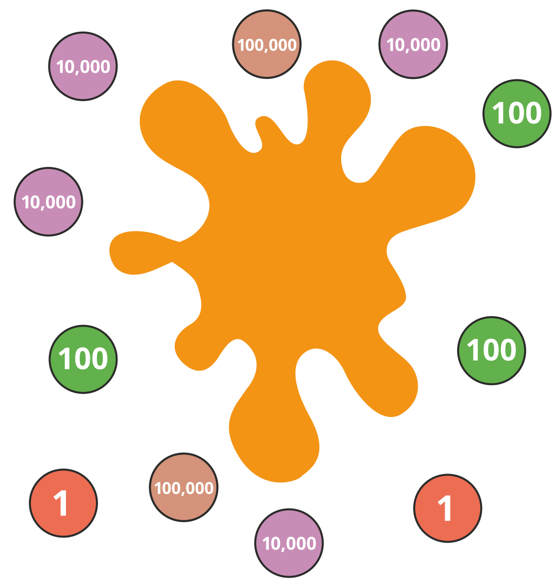
The rest are equal to 680,000

Some of the place value counters are hidden.

The total value of the counters is 265,312

What place value counters could be hidden?

Find at least three solutions.



any set of counters that add up 25,010

multiple possible answers, e.g.

- two 10,000 counters, five 1,000 counters and one 10 counter
- twenty-five 1,000 counters and one 10 counter
- two 10,000 counters, three 1,000 counters, twenty 100 counters and one 10 counter

Number line to 1,000,000

Notes and guidance

This step begins with a recap of number lines to 10,000, before moving on to explore number lines up to 100,000 and 1,000,000

Children label partially completed number lines, identify points labelled on number lines and show where a given number would lie on a number line. They look at both the exact placement of multiples of 10,000 or 100,000 and the approximate placement of numbers such as 245,678

Recognising the value of the midpoint between two multiples on a number line is key to their understanding and will support the use of number lines when rounding numbers in later steps.

Things to look out for

- Where number lines have more than one set of divisions, children may mix up the intervals between large divisions and smaller divisions.
- Children may confuse the number of intervals and the number of divisions.
- Children may not use the correct multiples when looking at midpoints, for example thinking the midpoint between 1,000 and 2,000 is 1,005

Key questions

- What are the values at the start and the end of the number line?
- How many large intervals are there in the whole number line? What is each large interval worth?
- How many small intervals are there between each of the large intervals on the number line? What is each small interval worth?
- What is the midpoint between ______ and ______?

Possible sentence stems

- The difference in value between the start and end point is ______
- There are ______ intervals.
- The number line is counting up in ______

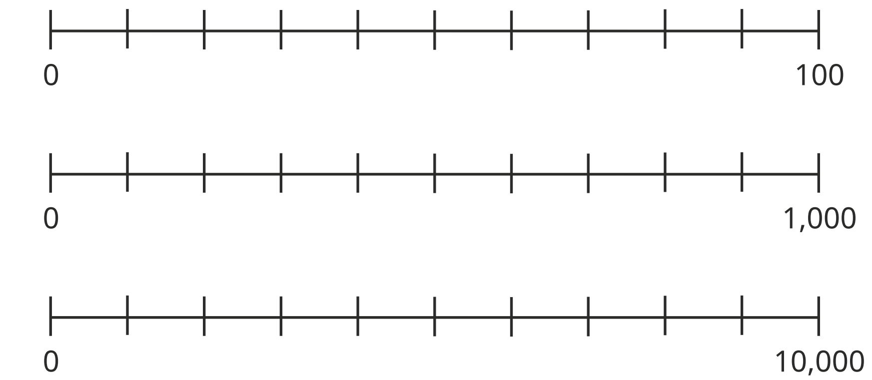
National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000

Number line to 1,000,000

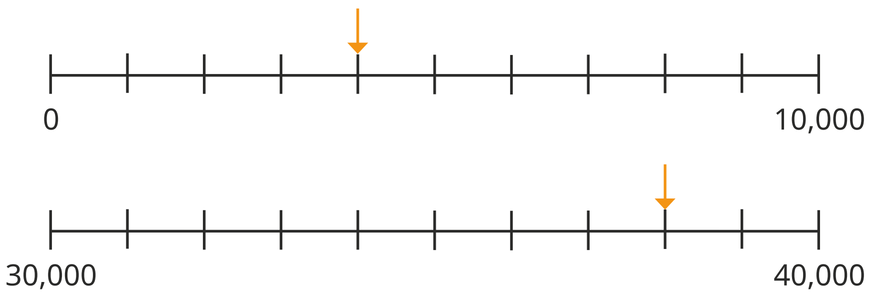
Key learning

- Label the number lines.

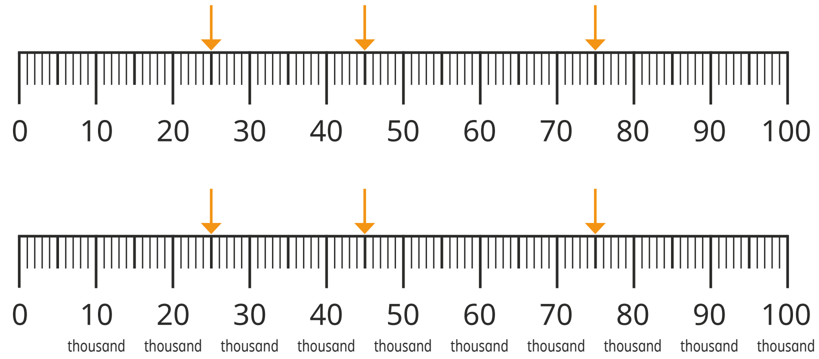


What is the same? What is different?

- What numbers are the arrows pointing to?

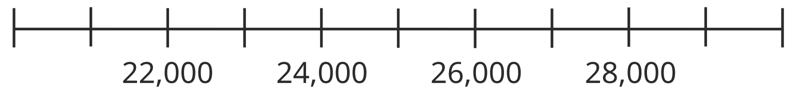


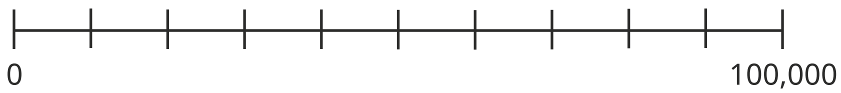
- What numbers are the arrows pointing to?



What is the same about the number lines? What is different?

- Label the start and end points on the number line.



- 

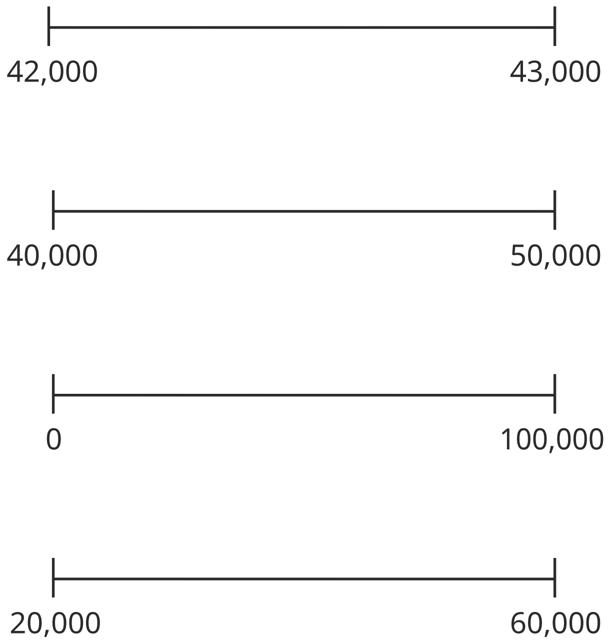
Draw arrows on the number line to show:

- the exact position of 60,000
- the approximate position of 35,000
- the approximate position of 82,369

Number line to 1,000,000

Reasoning and problem solving

Estimate the position of 42,500 on each number line.

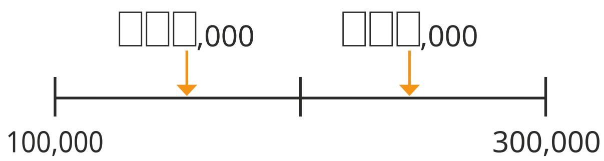


Explain your method.

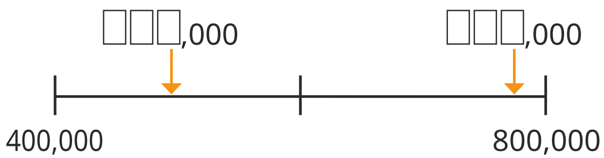
Children should draw an arrow to the approximate position on each number line and be able to explain their reasoning.

Use the digit cards to complete the labels on the number lines.

1 2 3 4 5 6



4 5 6 7 8 9



multiple possible answers, e.g.

156,000 and 243,000

496,000 and 785,000

Compare and order numbers to 100,000

Notes and guidance

In this small step, children build on their learning of comparing and ordering numbers in earlier years to compare and order numbers up to 100,000

They can use a variety of representations to help them, such as place value counters, place value charts and number lines, but the main focus of the step is to compare and order using the place value of the digits within the numbers. Children first compare pairs of numbers and then move on to ordering sets of three or more numbers.

This small step provides an opportunity to revisit previous learning from this block, as children could be asked to compare and order numbers that are written in Roman numerals.

Things to look out for

- Children may only look at the digits and not consider the place value of the digits within the numbers.
- Where numbers have a different number of digits, children may only look at the first digit.
- Children often confuse the inequality symbols and their meanings.

Key questions

- Which digit in each number has the greatest value? What are the values of these digits?
- When comparing two numbers with the same number of digits, if their first digits are equal in value, what do you look at next?
- What is the difference between ascending and descending order?
- What is different about comparing numbers with the same number of digits and comparing numbers with different numbers of digits?

Possible sentence stems

- The first place value column I need to look at is ______
- ______ is greater/less than ______, so ______ is greater/less than ______

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit

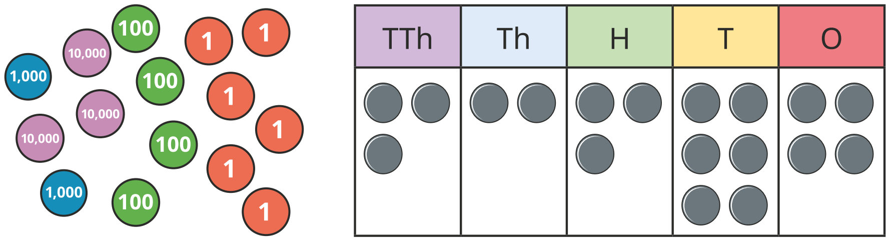
Compare and order numbers to 100,000

Key learning

- Identify the greater number in each pair.
  - 63 and 68
  - 63,000 and 68,000
  - 63,912 and 68,002

  What is the same and what is different?

- Which is the greater number?



- Write <, > or = to make the statements correct.

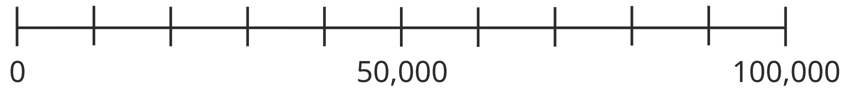
  45,000 ◯ 54,000

  10,160 ◯ 9,999

  65,000 ◯ 60,700

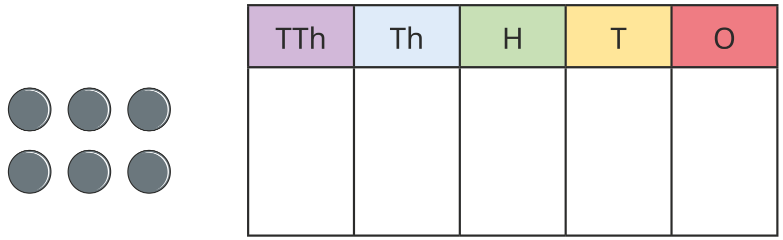
- Put the numbers in order, starting with the smallest.

  You can use the number line to help you.



  6,400 60,400 46,000 40,600 20,600 6,004

- Use six counters to make five different 5-digit numbers.



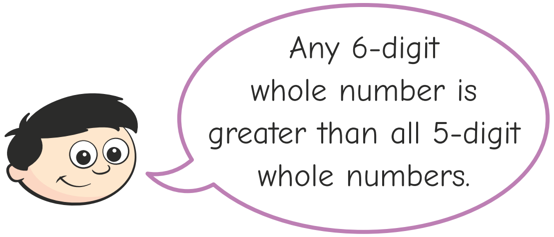
  Order your numbers from greatest to smallest.

- Write the numbers in ascending order.

  34,706 | MMMDXL | 3,099 | 5,000 more than thirty thousand | thirty-three thousand and thirty-three

Compare and order numbers to 100,000

Reasoning and problem solving



Do you agree with Dexter?

Explain your answer.

Yes

Here are six digit cards.

7	2	5	1	9	3

Using five of the digits, what is the greatest number you can make?

Using all six digits, what is the smallest number you can make?

97,532

123,579

Use the digit cards to make three different 5-digit numbers that match the clues.

0	1	2	3	4
5	6	7	8	9

- The digit in the ones column and the digit in the hundreds column have a difference of 2
- The digit in the hundreds column and the digit in the ten-thousands column have a difference of 2
- The sum of all the digits in the number is 19

Write your numbers in ascending order.

multiple possible answers, e.g.

18,325

47,260

56,341

Compare and order numbers to 1,000,000

Notes and guidance

In this small step, children build on the previous step to compare and order numbers up to 1,000,000

The representations used previously can continue into this step; however, the focus will shift more towards number lines as they are more efficient when representing numbers of increasing value.

Encourage children to make connections between the position of numbers on a number line and their value. They should recognise that when working on horizontal number lines, numbers further to the right have a greater value. Word problems involving real-world examples, such as comparing populations, are also introduced.

Things to look out for

- Children may only look at the digits and not consider the place value of the digits within the numbers.
- Children may need to be reminded of the meanings of the inequality symbols as well as the words "ascending" and "descending".
- Placeholders can cause difficulty when working with larger numbers.

Key questions

- Which digit in each number has the greatest value? What are the values of these digits?
- When comparing two numbers with the same number of digits, if their first digits are equal in value, what do you look at next?
- What is the difference between ascending and descending order?
- What is different about comparing numbers with the same number of digits and comparing numbers with different numbers of digits?

Possible sentence stems

- The first place value column I need to look at is ______
- ______ is greater/less than ______, so ______ is greater/less than ______

National Curriculum links

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit

Compare and order numbers to 1,000,000

Key learning

- Identify the greater number in each pair.
  - 59 51
  - 59,000 51,000
  - 590,000 510,000

  What is the same and what is different?

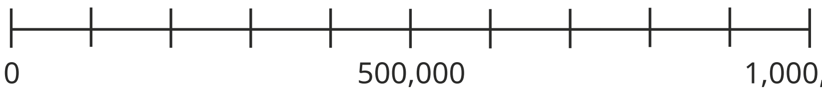
- Write <, > or = to make the statements correct.

  450,000 ◯ 540,000 101,600 ◯ 99,999

  650,000 ◯ 607,000 312,007 ◯ 312,070

- Put the numbers in ascending order.

  You can use the number line to help you.



  64,000 604,000 460,000 40,600 200,600 6,004

- The table shows the populations in some towns and cities in Yorkshire.

  List the towns and cities in descending order of population.

Town or city	Population
Halifax	88,134
Brighouse	32,360
Leeds	792,925
Huddersfield	146,234
Wakefield	343,932
Bradford	536,986

- Estimate the positions of the numbers on the number line.

  A four hundred and ten thousand
  B 95,770
  C half a million
  D eight hundred thousand

  Write the numbers in ascending order.

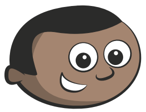
Compare and order numbers to 1,000,000

Reasoning and problem solving

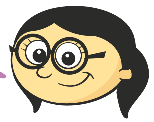
Here are four number cards.

101,080	one hundred thousand
one thousand one hundred	99,280

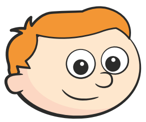
Mo, Annie and Ron each choose a card.



Mo: My number has the greatest value.



Annie: My number has 8 tens.



Ron: My number is greater than Annie's but less than Mo's.

Which card is left over?

one thousand one hundred

Write <, > or = to make the statements correct.

600,000 + 80,000 ◯ 618,000

10,000 less than 723,000 ◯ 722,000

999,999 ◯ one million

50,000 ◯ half a million

20 ten-thousands ◯ 200 thousands

>

<

<

<

=

Round to the nearest 10, 100 or 1,000

Notes and guidance

In this small step, children build on their knowledge of rounding to the nearest 10, 100 and 1,000 from Year 4, now also rounding numbers beyond 10,000 to these degrees of accuracy.

It is important that children hear and use the language of "rounding to the nearest" rather than "rounding up" and "rounding down", as this can lead to errors. Number lines are a particularly useful tool to support this, as children can see which multiples of 10, 100 or 1,000 the given numbers are closer to. It is worth discussing with children the convention that when there is a 5 in the relevant place value column, despite being exactly halfway between the two multiples, we round to the next one.

Things to look out for

- Children may not round to the correct degree of accuracy, for example rounding to the nearest 100 instead of the nearest 1,000
- Children may be confused by the language "round down"/"round up" and thus round 72,160 to 71,000 (or 71,160) when asked to round to the nearest 1,000
- Children may look at the thousands digit rather than the hundreds when rounding to the nearest 100

Key questions

- Which multiples of 10/100/1,000 does the number lie between?
- Which multiple on the number line is the number closer to?
- What is the number rounded to the nearest 10/100/1,000?
- Which place value column should you look at to round the number to the nearest 10/100/1,000?
- What happens when a number is exactly halfway between two numbers on a number line?

Possible sentence stems

- The previous multiple of 10/100/1,000 is ______
- The next multiple of 10/100/1,000 is ______
- ______ is closer to ______ than ______
- ______ rounded to the nearest 10/100/1,000 is ______

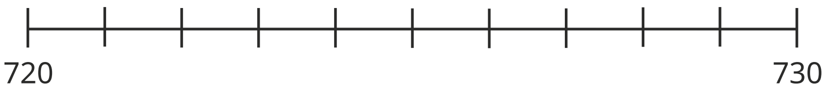
National Curriculum links

- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000

Round to the nearest 10, 100 or 1,000

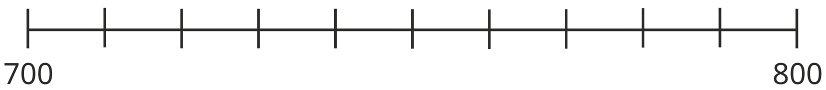
Key learning

- Mark the position of 728 on the number line.



Use the number line to round 728 to the nearest 10

Now estimate the position of 728 on this number line.



Use the number line to round 728 to the nearest 100

- Between which two multiples of 1,000 does the number 6,741 lie?

What is 6,741 rounded to the nearest 1,000?

- 3,500 is exactly halfway between 3,000 and 4,000

What is 3,500 rounded to the nearest 1,000?

- 8,317 people attend a pop concert.

Round the number of people at the concert to the nearest 10

Round the number of people at the concert to the nearest 100

Round the number of people at the concert to the nearest 1,000

- 31,409 people attend a football match.

Round the number of people at the match to the nearest 100

Round the number of people at the match to the nearest 1,000

- Eva runs every night for a week.

Altogether she runs 28,650 m.

Round the distance she runs to the nearest 100 m.

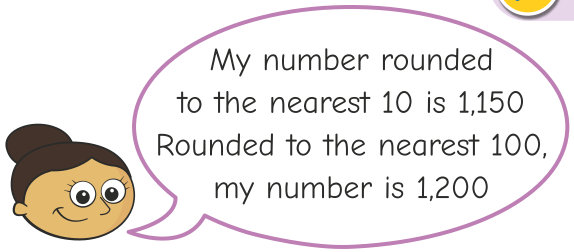
Round the distance she runs to the nearest kilometre.

- Which numbers round to 4,600 to the nearest 100?

4,620	4,605	4,590	4,545	4,499	4,650

Round to the nearest 10, 100 or 1,000

Reasoning and problem solving



Find all the possible whole number values of Dora's number.

1,150, 1,151, 1,152
1,153, 1,154

Explain why Tiny is wrong.

When rounding to the nearest 1,000, the answer must be a multiple of 1,000

The correct answer is 5,000

When rounded to the nearest 10, a number is 50

When rounded to the nearest 100, the number is zero.

Find all the possible whole number values of the number.

45, 46, 47, 48, 49

Mo is thinking of a number.

- The number is 5,000 when rounded to the nearest 1,000
- The number is also 5,000 when rounded to the nearest 100
- The number is also 5,000 when rounded to the nearest 10
- The number is not 5,000

What is the greatest possible value of the number?

5,004

Round within 100,000

Notes and guidance

In this small step, children build on their learning in the previous step to round any number within 100,000 to the nearest 10, 100, 1,000 or 10,000. Rounding to the nearest 10,000 is the new learning.

They should be confident with multiples of 10,000 from earlier steps in this block, and the process of rounding is also familiar. Children need to realise that the midpoint of two multiples of 10,000 ends in 5,000, so they need to look at the digit in the thousands column to determine how to round the number.

As in the previous steps, be careful with the language of “round up” and “round down” in case children mistakenly change the wrong digits when rounding.

Things to look out for

- Children may not look at the correct column to make their decisions about rounding, for example rounding 24,555 to 30,000 to the nearest 10,000 as they have misapplied the rule “5 or more rounds up”.
- Children may be confused by the language “round down”/”round up”, for example rounding 78,564 to 88,564 to the nearest 10,000

Key questions

- Which multiples of 10,000 does the number lie between?
- Which division on the number line is the number closer to?
- What is the number rounded to the nearest 10,000?
- Which place value column should you look at to round the number to the nearest 10/100/1,000/10,000?
- What happens if a number lies exactly halfway between two multiples of 10,000?

Possible sentence stems

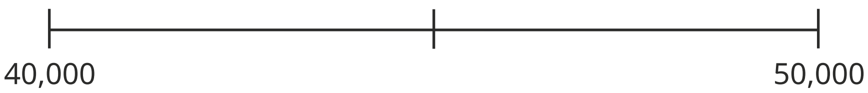
- The previous multiple of 10,000 is ______
- The next multiple of 10,000 is ______
- ______ is closer to ______ than ______
- ______ rounded to the nearest 10,000 is ______

National Curriculum links

- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000

Round within 100,000

Key learning

- 

  What number is halfway between 40,000 and 50,000?

  Draw an arrow to show the approximate position of 48,725 on the number line.

  Round 48,725 to the nearest 10,000

- Round 86 and 174 to the nearest 10

  Round 86,000 and 174,000 to the nearest 10,000

  What is the same and what is different?

- Round each number to the nearest 10,000

41,000	41,900
41,009	41,999
41,090	4,199

  What is the same and what is different?

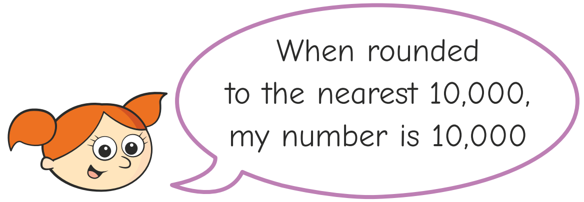
- The circumference of Earth is 24,901 miles.

  Round this distance to the nearest 1,000 miles.

  Round this distance to the nearest 10,000 miles.

  Which is the better approximation to use?

- Alex is thinking of a number.



  Which of these numbers could be Alex's number?

8,000	18,000	12,000	2,000	5,000
15,000	1,500	4,999	14,999	

  Explain how you know.

Round within 100,000

Reasoning and problem solving

Here is a newspaper headline about a football match.

Do you think exactly 60,000 people watched the football match?

What is the smallest number of people who watched the match, if the number in the headline has been:

- rounded to the nearest 10,000
- rounded to the nearest 1,000
- rounded to the nearest 100?

The headline is probably not an exact value.

55,000

59,500

59,950

The difference between two 5-digit numbers is 5

When the numbers are rounded to the nearest 1,000, the difference is 1,000

What could the numbers be?

any two 5-digit numbers with a difference of 5 where the last three digits are between 496 and 504, for example 52,498 and 52,503

By rounding both numbers to the nearest 10,000, estimate the answer to the calculation.

47,826 + 88,112

Is your estimate greater than or less than the actual answer?

How do you know?

140,000

---

greater

Round within 1,000,000

Notes and guidance

Building on the previous two steps, children now round any number up to 1,000,000 to any power of 10 up to 100,000. This is the first time that children round to the nearest 100,000

You may wish to practise counting in 100,000s first, and then practise rounding to the nearest 100,000 before looking at mixed questions.

It is worth discussing which approximations are most appropriate, for example why we would not give the population of a city to the nearest 10 or the population of a small town to the nearest 100,000

Things to look out for

- Children may not look at the correct column to make their decisions about rounding, for example rounding 245,555 to 300,000 to the nearest 100,000 as they have misapplied the rule "5 or more rounds to the next multiple".
- Children may be confused by the language "round down"/"round up", for example rounding 428,513 to 328,513 (or 300,000) to the nearest 100,000
- Children may not round to the required degree of accuracy, for example misreading "round to the nearest 100,000" as "round to the nearest 100".

Key questions

- Which multiples of 100,000 does the number lie between?
- How can you represent the rounding of this number on a number line?
- Which division on the number line is the number closer to?
- What is the number rounded to the nearest 100,000?
- What is the most appropriate way of rounding this number?
- What place value column should you look at to round the number to the nearest 10/100/1,000/10,000/100,000?

Possible sentence stems

- The previous multiple of 100,000 is ______
- The next multiple of 100,000 is ______
- ______ is closer to ______ than ______
- ______ rounded to the nearest 100,000 is ______

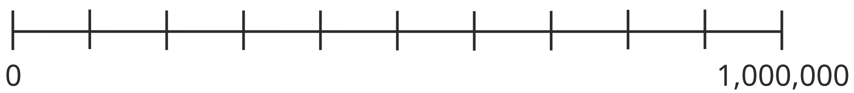
National Curriculum links

- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000

Round within 1,000,000

Key learning

- Complete the number line.



Between which two multiples of 100,000 does 735,292 lie?

Round 735,292 to the nearest 100,000

- The table shows the masses of some famous statues.

Statue	Mass
Statue of Liberty	201,400 kg
Christ the Redeemer	635,000 kg
Spring Temple Buddha	987,000 kg
Mustang Stone Buddha	58,000 kg

Round the mass of each statue to the nearest 10,000 kg.

Round the mass of each statue to the nearest 100,000 kg.

- The average distance of the Moon from Earth is 384,389 km.

Round this distance to the nearest 1,000 km.

Round this distance to the nearest 10,000 km.

Round this distance to the nearest 100,000 km.

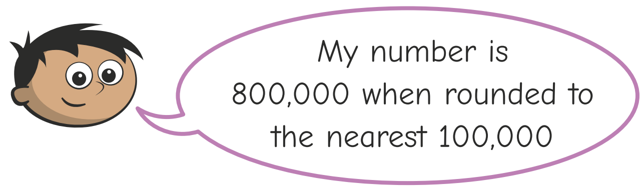
Which do you think is the most appropriate number to round the distance to?

- The greatest ever attendance at a football match was the World Cup final between Brazil and Uruguay in 1950

173,850 people watched the game.

Round this number to the nearest 1,000, 10,000 and 100,000

Which do you think is the most appropriate number to round the attendance to?

- 

What is the greatest integer Amir could be thinking of?

What is the smallest integer Amir could be thinking of?

Round within 1,000,000

Reasoning and problem solving

The difference between two 5-digit numbers is 200

When each number is rounded to the nearest 100,000, the difference between them is 100,000

What could the two numbers be?

Find all the possible answers.

49,900 and 50,100

49,800 and 50,000

328,154 people buy tickets for a festival.

Tickets are printed in batches of 10,000

How many batches of tickets should the organisers print?

33 batches

A, B and C are three different whole numbers.

- When the difference between A and B is rounded to the nearest 100, the answer is 700
- When the difference between B and C is rounded to the nearest 100, the answer is 400
- None of the numbers are multiples of 10

Find a possible set of values for A, B and C.

Compare answers with a partner.

Are your values of A, B and C in the same order greatest to smallest?

Are your differences smaller or greater?

A – B (or B – A) is between 650 and 749

B – C (or C –B) is between 350 and 449

multiple possible answers, e.g.

A = 1,199
B = 450
C = 1

A = 651
B = 1
C = 351

Autumn Block 2

Addition and subtraction

Small steps

Step 1	Mental strategies
Step 2	Add whole numbers with more than four digits
Step 3	Subtract whole numbers with more than four digits
Step 4	Round to check answers
Step 5	Inverse operations (addition and subtraction)
Step 6	Multi-step addition and subtraction problems
Step 7	Compare calculations
Step 8	Find missing numbers

Mental strategies

Notes and guidance

In this small step, children recap and build on their learning from previous years to mentally calculate sums and differences using partitioning. They use their knowledge of number bonds and place value to add and subtract multiples of powers of 10. Children unitise to help them complete a calculation. For example, if they know that 3 + 5 = 8, then 3 thousand + 5 thousand = 8 thousand and 3,000 + 5,000 = 8,000

Children also count forwards and backwards in multiples of powers of 10 to answer questions such as 1,050 – 100 without the need for a formal written method.

Children explore strategies such as compensation and adjustment to mentally calculate the answer to questions such as 14,352 + 999 or 14,352 – 999. This helps them to make connections between calculations and will be developed further in Year 6

Things to look out for

- Children need to be fluent in their knowledge of number bonds to support the mental strategies.
- Children may opt to use a formal method even when this is time-consuming and/or inappropriate.

Key questions

- How does knowing that 2 + 5 = 7 help you to work out 20,000 + 50,000?
- How can the numbers be partitioned to help add/subtract them?
- Are any of the numbers multiples of powers of 10? How does this help you to add/subtract them?
- What number is 999 close to? How does that help you to add/subtract 999 from another number?

Possible sentence stems

- The sum of ______ ones and ______ ones is ______ ones, so the sum of ______ thousands and ______ thousands is ______ thousands.
- I can partition the number into ______ , ______ and ______ and add the parts separately.

National Curriculum links

- Add and subtract numbers mentally with increasingly large numbers

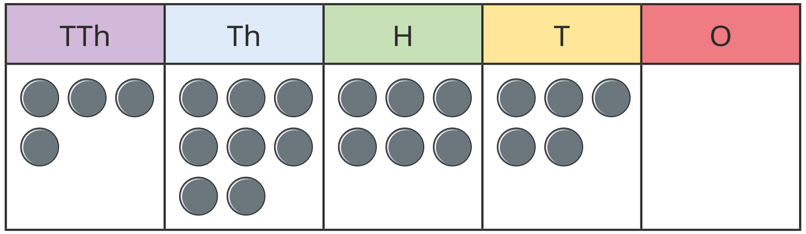
Mental strategies

Key learning

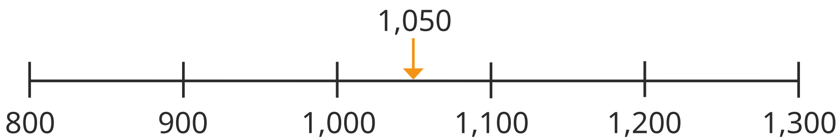
- Use the fact that 8 + 4 = 12 to work out the additions.
  - 8,000 + 4,000
  - 800 + 400
  - 80,000 + 40,000

- Find the sum of each pair of numbers.
  - 300,000 and 400,000
  - 62,000 and 6,000
  - 110,000 and 230,000
  - 5,020 and 9,060

- Use the place value chart to help you work out the subtractions.

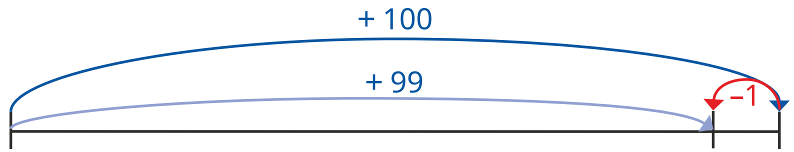


  - 48,650 – 3,000
  - 43,650 – 200
  - 43,650 – 10
  - 48,650 – 3,210
  - 48,650 – 7,100
  - 48,650 – 5,030

- 

  Use the number line to help you work out the calculations.
  - 1,050 + 100
  - 1,050 – 100

- The number line shows a method for adding 99 mentally.



  Use the number line to help you add 99 to 687

  Use a similar number line to help you subtract 99 from 687

- Work out the calculations.

3,724 + 999	3,724 – 999
3,724 + 990	3,724 – 990

- Work out the calculations.

46 + 29	460 + 290	460 + 299
59 + 59	590 + 590	599 + 599

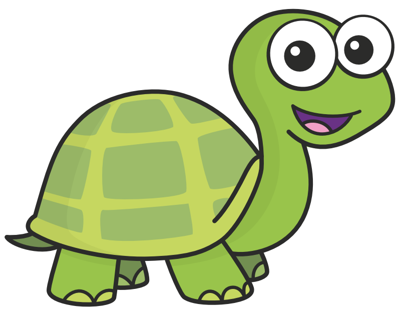
  What mental strategies did you use?

Mental strategies

Reasoning and problem solving

Tiny is using mental strategies to add numbers.

I know
800 + 300 = 1,100,
so 4,826 + 300 = 41,126

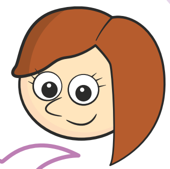


Explain why Tiny is wrong.

Find the correct answer to 4,826 + 300

5,126

Rosie is working out a subtraction.



1,000 – 372 = 999 – 371

Explain why Rosie is correct.

Work out the answer to 1,000 – 372

Use this strategy to work out the subtractions.

1,000 – 625

10,000 – 6,832

100,000 – 47,356

628

375

3,168

52,644

Add whole numbers with more than four digits

Notes and guidance

In this small step, children revisit the use of the column method for addition and learn to apply this method to numbers with more than four digits.

A range of representations can be used for support in this step, including place value counters and place value charts. These representations are particularly useful when performing calculations that require an exchange. Children may find it easier to work with squared paper and labelled columns as this will support them in placing the digits in the correct columns, especially with figures containing different numbers of digits.

If appropriate, children could practise their rounding skills to estimate the answer before working out the calculation, and then use it as a sense-check for their solution. This skill is covered in detail later in this block.

Things to look out for

- Children may not line up the numbers in the columns correctly.
- Children may write the exchanged digits in the wrong column(s).
- Children who are not secure in their number bonds may make numerical errors within columns.

Key questions

- Does it matter which number goes at the top when using the column method?
- Will you need to make an exchange? Which columns will be affected if you do? How do you know?
- Does it matter if the numbers have different numbers of digits?
- How do you know which digits to "line up" in the calculation?
- How do you know if the calculation is an addition?

Possible sentence stems

- In column addition, we start from the place value column that has the ______ value.
- The ______ is in the ______ column. It represents ______

National Curriculum links

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Add whole numbers with more than four digits

Key learning

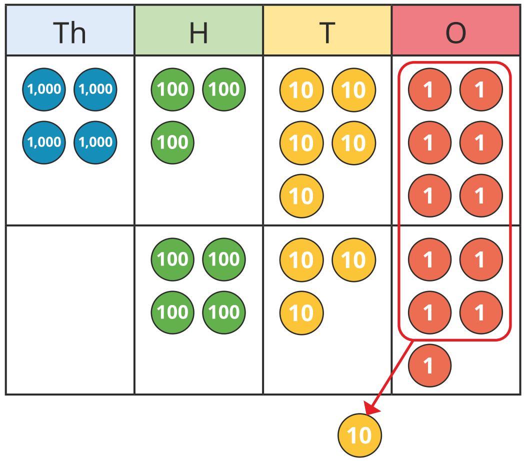
- Use the column method to work out the additions.

		4	7	
	+	3	8	

		2	4	7	
	+	5	3	8	

		3	6	4	7	
	+	4	9	2	8	

- Ron uses place value counters to calculate 4,356 + 435



		4	3	5	6	
	+		4	3	5	
		4	7	9	1	
				1		

Use Ron's method to work out the additions.

32,461 + 4,352

48,276 + 5,613

- Jack, Kim and Eva are playing a computer game.
  - Jack has 3,452 points.
  - Rosie has 4,039 points.
  - Eva has 10,989 points.

  How many points do Jack and Rosie have altogether?

  How many points do Rosie and Eva have altogether?

  How many points do Jack and Eva have altogether?

  How many points do Jack, Rosie and Eva have altogether?

- Find the sum of seventy-three thousand, five hundred and eighty-four and twenty-eight thousand, nine hundred and nine.

- Find the answers to the calculations.

  In each case decide whether a mental method or written method is more appropriate.

12,724 + 43,610

63,800 + 2,002

9,999 + 8,712

313,500 + 89,019

Add whole numbers with more than four digits

Reasoning and problem solving

Work out the missing numbers.

	☐	4	☐	3	☐
+	2	☐	5	☐	2
	7	8	5	2	9

54,937
+ 23,592
78,529

What mistake has been made?

1,562 + 301 = 4,572

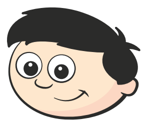
The incorrect place value has been assigned to each digit in 301

Dexter is estimating the sum of a 6-digit number and a 5-digit number.

My 6-digit number rounds to 200,000 to the nearest 10,000

I have rounded my 5-digit number to the nearest 1,000

My estimate of the total is one-quarter of a million.



What could Dexter's numbers be?

What is the greatest possible total of Dexter's numbers?

What is the smallest possible total of Dexter's numbers?

6-digit number: between 195,000 and 204,999

5-digit number: between 49,500 and 50,499

255,498

244,500

Subtract whole numbers with more than four digits

Notes and guidance

In this small step, children revisit the use of the column method for subtraction and learn to apply this method to numbers with more than four digits.

A range of representations can be used for support in this step, including place value counters and place value charts. These representations are particularly useful when performing calculations that require an exchange. Children may find it easier to work with squared paper and labelled columns as this will support them in placing the digits in the correct columns, especially with figures containing different numbers of digits.

Children should experience both questions and answers where zero appears in columns as a placeholder.

Things to look out for

- Children may always subtract the smaller digit from the larger digit instead of making an exchange when needed.
- The need for repeated exchanges may cause difficulty.
- When using the column method, children may arrange the numbers incorrectly.

Key questions

- Which number goes at the top when using the column method? Does this affect the final answer?
- Will you need to make an exchange? Which columns will be affected if you do? How do you know?
- Does it matter if the numbers have different numbers of digits?
- How do you know which digits to "line up" in the calculation?
- How do you know if the calculation is a subtraction?

Possible sentence stems

- In column subtraction, we start from the place value column that has the ______ value.
- There are not enough ______, so I need to exchange 1 ______ for 10 ______

National Curriculum links

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Subtract whole numbers with more than four digits

Key learning

- Use the column method to work out the subtractions.

	8	4
–	3	6

	6	3	2
–	4	1	7

	4	6	8
–	2	9	3

	3	1	2	5
–	2	4	1	7

- Work out the subtraction.

  Use the place value chart and the column method to help you.

Tth	Th	H	T	O
10,000 10,000 10,000 10,000	1,000 1,000 1,000 1,000 1,000	100 100 100 100 100	10 10 10	1 1 1 1 1 1

	4	5	5	3	6
–		8	4	2	6

- There are 43,662 fans at a football match.

  31,547 of the fans are adults.

  How many of the fans are not adults?

- The population of Hereford is 63,689

  The population of Chester is 87,593

  Find the difference between the population of Hereford and the population of Chester.

- Subtract twelve thousand, three hundred and seventy from eighteen thousand, one hundred and twenty-four.

- Find the answers to the calculations.

  In each case, decide whether a mental method or written method is more appropriate.

12,000 – 2	46,312 – 15,000
35,295 – 16,359	90,000 – 23,518

Subtract whole numbers with more than four digits

Reasoning and problem solving

Work out the missing numbers.

		5		4		8	
	–		1		2		
		2	0	8	5	8	

52,478
– 31,620
20,858

623 + 754 = 1,377

Use the calculation to complete the number sentences.

______ – 754 = 623

______ – 6,230 = 7,540

137,700 – 75,400 = ______

1,377

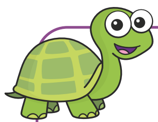
13,770

62,300

Work out the missing numbers.

		6		2		6	
	–		8		7		
		3	6	7	4	4	

65,216
– 28,472
36,744



Tiny is working out a subtraction.

53,209 – 27,452 = 34,257

What mistake has Tiny made?

Instead of making exchanges, Tiny has found the difference between the digits in each place value column.

Round to check answers

Notes and guidance

In this small step, children practise rounding in order to estimate the answers to both additions and subtractions. They also review mental strategies for estimating answers.

Children should be familiar with the word "approximate", and the degree of accuracy to which to round is a useful point for discussion. Generally, rounding to the nearest 100 for 3-digit numbers, the nearest 1,000 for 4-digit numbers and so on is appropriate, but there is no need to formally introduce the language and idea of "rounding to one significant figure" at this stage.

Children may need reminding that the reason we round in this context is to produce a calculation that can easily be completed mentally.

Things to look out for

- Children may need support in deciding to what degree of accuracy they should round given numbers.
- If children have any difficulties or misconceptions with rounding this will hold them back when estimating.
- Children may forget to compare their answers with their estimates.

Key questions

- Which multiples of ______ does the number lie between?
- Which division on the number line is the number closer to?
- What is the number rounded to the nearest ______?
- What place value column should we look at to round the number to the nearest 10/100/1,000/10,000/100,000?
- How could you use your estimates to check your answers?
- Is the actual answer going to be greater or less than your estimate? Why?

Possible sentence stems

- The previous multiple of ______ is ______
- The next multiple of ______ is ______
- ______ rounded to the nearest ______ is ______
- The approximate answer is ______

National Curriculum links

- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000
- Add and subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

Round to check answers

Key learning

- Round the numbers to find an estimate of the answer to 6,789 + 2,870

  6,789 rounded to the nearest 1,000 is ______

  2,870 rounded to the nearest 1,000 is ______

  The estimated total is ______ + ______ = ______

  Compare the estimate with the actual answer.

- Round each number to the nearest 100,000 to estimate the answers to the calculations.

517,000 + 289,000	517,000 – 289,000
126,539 + 723,628	809,375 – 610,005

- Annie estimates the answer to 22,223 + 5,867 by rounding both numbers to the nearest 1,000

  Jack estimates the answer to 22,223 + 5,867 by rounding both numbers to the nearest 10,000

  Compare Annie's method with Jack's method.

  Work out the actual answer. Which estimate was closer?

- The table shows the number of tickets sold by an airline during a three-month period.

Month	Tickets sold
February	18,655
March	31,402
April	27,092

  - Work out the total number of tickets sold in February and March.

    Use an estimate to check your answer.

  - The approximate total number of tickets sold in April and May was 50,000

    Estimate the number of tickets sold in May.

- Mrs Khan wants to buy a laptop, a monitor and a keyboard.

Mrs Khan has £1,700

Estimate whether she can afford all three items.

Round to check answers

Reasoning and problem solving

Mo has completed an addition.

31,207 + 6,529 = 96,497

Use an estimate to show that Mo must have made a mistake.

30,000 + 7,000

= 37,000

Mo's answer is too big.

When two numbers are rounded to the nearest 10,000, their sum is 100,000

What could the numbers be? Discuss possible answers with a partner.

What is the smallest possible actual total of the numbers?

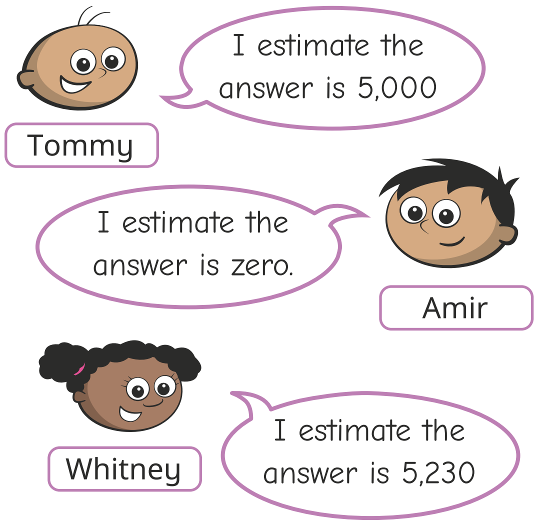
What is the greatest possible actual total of the numbers?

90,000 (e.g. 65,000 + 25,000)

109,998 (e.g. 54,999 + 54,999)

Tommy, Amir and Whitney are working out a subtraction.

64,942 – 59,713



Explain why the children all have different estimates.

Work out the actual answer.

Whose estimate is most accurate?

They rounded the numbers to different powers of 10

5,229

Whitney's

Inverse operations (addition and subtraction)

Notes and guidance

Children should know that addition and subtraction are inverse operations from learning in earlier years, and should already be aware that addition is commutative and subtraction is not.

Children can use bar models or part-whole models to establish families of facts that can be found from one calculation and then use inverse operations to check the accuracy of their calculations.

Children also use inverse operations to find unknown numbers, solving problems such as "I think of a number and add/subtract ______". This lays the groundwork for solving equations in Year 6 and beyond.

Things to look out for

- Children may see addition and subtraction as separate operations and not appreciate the connection between them.
- Children may think that subtraction is commutative.
- Children may need support to see the correct order in which to perform a subtraction to check a given addition.
- When solving "I think of a number" problems, children may use the given operation instead of the inverse operation.

Key questions

- If I add a number to another to get a total, what do you need to do to the total to find my original number?
- If I subtract a number from another to find the difference, what do you need to do to the difference to find my original number?
- What does an inverse operation do?
- What operation is the inverse of addition?
- What operation is the inverse of subtraction?

Possible sentence stems

- The inverse of ______ is ______
- To check that I have added/subtracted ______ correctly, I need to ______

National Curriculum links

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Inverse operations (addition and subtraction)

Key learning

- Write two additions and two subtractions shown by the bar model.

647	
289	358

- Aisha works out an addition.

  65 + 78 = 143

  Which subtractions can be used to check Aisha's addition?

  143 – 78 | 78 – 65 | 143 – 65 | 78 – 143

- Complete the bar model.

437	261

  Check your answer using a subtraction.

- Huan thinks of a number.

  He adds 17 to his number and gets the answer 40

  Which calculation can be used to find Huan's number?

  17 + 40 | 17 – 40 | 40 – 17 | 40 + 17

- Esther and Brett are playing a computer game.

  Esther scores 8,524 points.

  The total of both their scores is 19,384

  How many points did Brett score?

- Dani thinks of a number.

  After she adds 5,241 and subtracts 352, her new number is 9,485

  What was Dani's original number?

- Find the missing numbers.
  - 654 + ______ = 837
  - ______ – 719 = 424
  - 3,820 = 5,260 – ______
  - 19,456 = 2,345 + ______

  Use inverse operations to check your answers.

Inverse operations (addition and subtraction)

Reasoning and problem solving

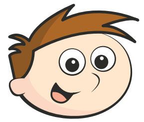
In the number pyramid, each number is the sum of the two numbers below.

Use addition and subtraction to complete the pyramid.

		14,031		
	6,976	6,415	7,616	
4,946	3,172		2,611	

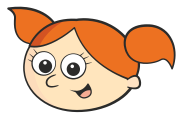
55,907				
28,485	27,422			
15,094	13,391	14,031		
8,118	6,976	6,415	7,616	
4,946	3,172	3,804	2,611	5,005

Teddy and Alex are each thinking of a number.



Teddy

98 less than my number is 465



99 more than my number is 465

Alex

Find the difference between Teddy's number and Alex's number.

98 + 99 = 197

Multi-step addition and subtraction problems

Notes and guidance

In this small step, children apply the strategies they have learned so far in this block to solve addition and subtraction problems with more than one step.

Children choose the operations needed at each step and then perform the calculations using an appropriate mental or written method. Problems are presented in both word form and with models. The use of bar models can help children to illustrate problems of this kind. While the models will not perform the calculation, they will help children to decide what operations are needed and why.

Although the focus is on addition and subtraction, sometimes division will be needed to find some of the numbers. The previous small step can also be reinforced by using inverse operations or approximations to check if answers are correct.

Things to look out for

- Children may find it difficult to interpret word problems, particularly if the context is unfamiliar.
- Children may choose the wrong operation.
- Commonly used words such as "more" can cause confusion as children assume this always means an addition is necessary.

Key questions

- What is the key information in the question?
- What can you work out straight away? How does this help you to answer the question?
- How can you represent this problem using a bar model? Which bar will be longer? Why?
- Do you need to add or subtract the numbers at this stage? How do you know?
- How can you check your answer?

Possible sentence stems

- The first step in solving the problem is …
- When I know ______, I can then ______
- To check my answer, I can …

National Curriculum links

- Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Multi-step addition and subtraction problems

Key learning

- Filip is writing a report.

  He writes the first 460 words on Monday and another 735 words on Tuesday.

  The report must be at least 2,500 words long.

  How many more words does Filip need to write?

- Year 5 and Year 6 are going on a school trip.

  The school has a bus with 56 seats and a minibus with 17 seats.

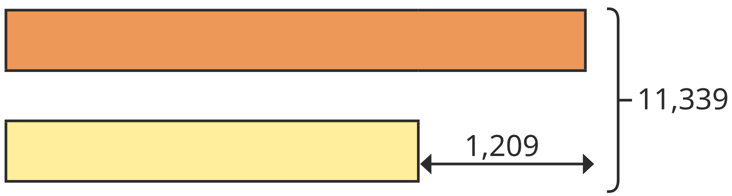
  There are 44 people in the Year 5 group and 38 people in the Year 6 group.

  How many more seats are needed for both groups to go on the trip?

- The sum of two numbers is 11,339

  The difference between the numbers is 1,209

  Use the bar model to help you find the two numbers.



- Mr Rose is buying items for his home.

  He has a budget of £1,500

  He buys a washing machine and a tumble dryer.

  Does he have enough money left to buy the dishwasher?

- A pole is used to measure the depth of water in a river.

  The part of the pole above the water is 95 cm.

  The part of the pole in the water is 35 cm greater than the part of the pole above the water.

  How long is the pole?

- Annie opens a book and sees two numbered pages.

  The sum of the page numbers is 317

  What is the number of the next page in the book?

Multi-step addition and subtraction problems

Reasoning and problem solving

A milkman has 250 bottles of milk.

During the day, he collects another 160 from the dairy and delivers 375 bottles.

Nijah works out how many bottles are left.

375 − 250 = 125

125 + 160 = 285

Do you agree with Nijah?

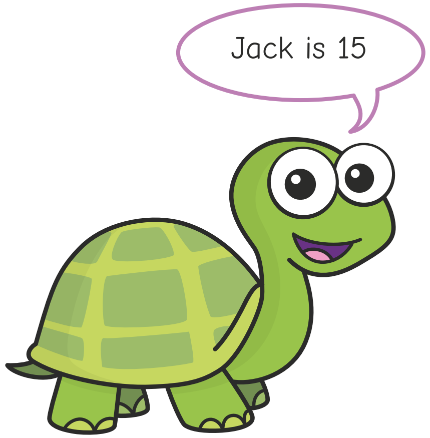
Explain your answer.

No

Mo is twice as old as Jack.

Dora is 3 years younger than Jack.

The sum of all their ages is 33



Explain the mistake Tiny has made.

How old is Jack?

9 years old

Compare calculations

Notes and guidance

In this small step, children are required to compare calculations. The focus is not on completing calculations, but instead exploring their structure in order to make a comparison. Children should understand the effect that adding to or subtracting from numbers in a calculation has on the answer to that calculation.

Bar models are a useful way of illustrating the relationships between calculations. It may be appropriate to concentrate on comparisons with 2-digit and 3-digit numbers before moving on to larger numbers.

The understanding children develop in this step will support them in the next step where they use a given fact to derive other answers. They also look at similar strategies for multiplication and division in future blocks.

Things to look out for

- When given calculations, children may automatically start to work out the answers rather than use strategies to make comparisons.
- When comparing calculations, children may not recognise two identical numbers if presented in a different order either side of the inequality symbol, for example 423 + 650 < 729 + 423

Key questions

- What is the same and what is different about the numbers in the two calculations?
- Which digits have changed and which have stayed the same?
- How will the answer change if you increase one of the numbers by ______?
- How will the answer change if you decrease one of the numbers by ______?
- How will the answer change if you increase/decrease both of the numbers by ______?

Possible sentence stems

- If I add/subtract ______ to/from one of the numbers in the calculation, the answer will change by ______
- If I add/subtract ______ to/from both of the numbers in the calculation, the answer will change by ______

National Curriculum links

- Add and subtract numbers mentally with increasingly large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Compare calculations

Key learning

- Which calculation has the greater answer, 983 + 410 or 983 + 510?

983	410
983	510

  Use the bar model to explain your answer.

- Which calculation has the greater answer?

  983 – 410   983 – 510

  How do you know?

- Write >, < or = to complete the calculations.

  47 + 28 ◯ 37 + 28

  64 + 91 ◯ 91 + 64

  651 – 286 ◯ 651 – 283

- Which calculation has the greatest answer?

  9,825 + 4,126   9,825 + 4,326   9,925 + 4,126

- Which calculations have an answer greater than the answer to 478 + 217?

  478 + 218   479 + 217   479 + 218   477 + 317

  Explain your answers.

- Which calculations have an answer greater than the answer to 5,618 – 3,257?

  5,618 – 4,257   5,618 – 1,257   5,619 – 3,257   6,618 – 3,257

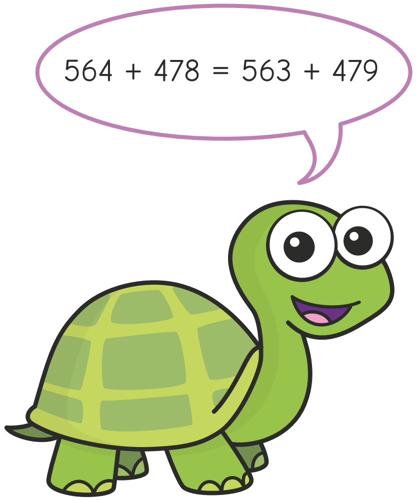
  Explain your answers.

- Put the addition cards in order of size, starting with the one with the greatest answer.

  463 + 127   563 + 327   483 + 127   463 + 327   463 + 337

Compare calculations

Reasoning and problem solving



Explain why Tiny is correct.

Which of the calculations have the same answer as 564 – 478?

565 – 479	563 – 479
565 – 477	563 – 477

563 is 1 less than 564 and 479 is 1 more than 478, so the total does not change.

565 – 479

563 – 477

16,853 + 23,671 = 40,524

Use the addition to work out these calculations.

- 16,953 + 23,671
- 16,883 + 23,691
- 40,524 – 16,853
- 42,524 – 16,853
- 40,524 – 17,853
- 405,240 – 236,710

Compare methods with a partner.

40,624

40,574

23,671

25,671

22,671

168,530

Find missing numbers

Notes and guidance

This small step begins with revision of the use of inverse operations to find a missing number in a calculation. Children then build on the previous small step to solve missing number problems by comparing calculations.

Children need to understand that when two numbers are increased by the same amount the difference remains the same, and that the total of two numbers remains the same if one number has been increased by an amount and the other decreased by the same amount. Bar models and/or number lines can be used to illustrate these and other related concepts.

Children could be encouraged to revisit rounding to estimate and approximate as a way of sense-checking their answers.

Things to look out for

- Children may mix up the different effects on additions and subtractions if one or more of the numbers is adjusted.
- Children may try to find the missing number by performing a long series of calculations instead of looking at the relationships between the numbers in the given calculations.

Key questions

- What is the same and what is different about the numbers in the two calculations?
- If the two additions/subtractions have the same result, what does that tell you about the numbers in the additions/subtractions?
- If you increase/decrease the first number by ______ , what do you need to do to the second number for the total/difference to stay the same?

Possible sentence stems

- ______ has been added/subtracted to/from the first number, so ______ must be ______ to/from the second number to keep the total the same.
- ______ has been added/subtracted to/from the first number, so ______ must be ______ to/from the second number to keep difference the same.

National Curriculum links

- Add and subtract numbers mentally with increasingly large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Find missing numbers

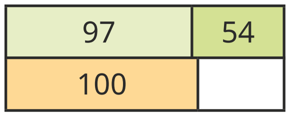
Key learning

- Complete the calculations.
  - ______ – 100 = 5,823
  - ______ – 1,000 = 5,823
  - 5,423 + ______ = 5,823
  - 3,623 + ______ = 5,823

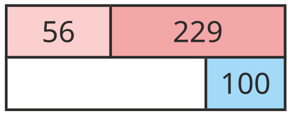
- Complete the calculations.

  Use the bar models to help you.

  97 + 54 = 100 + ______



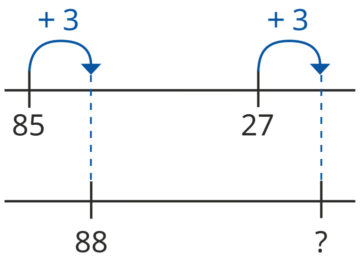
  56 + 229 = ______ + 100



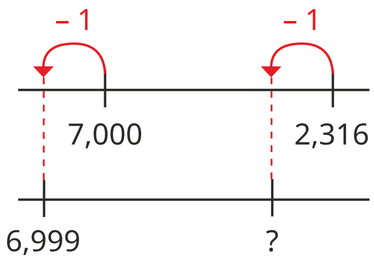
- Complete the calculations.

  Use the number lines to help you.

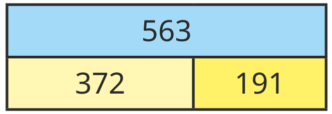
  85 – 27 = 88 – ______

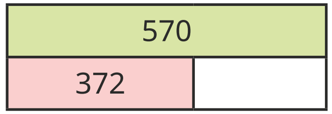


  7,000 – 2,316 = 6,999 – ______



- Use the first bar model to work out the missing number in the second bar model.





- Complete the calculations.
  - 536 + 275 = 540 + ______
  - 536 – 275 = 540 – ______
  - 3,000 – 513 = 2,999 – ______
  - 2,685 + ______ = 2,695 + 3,541

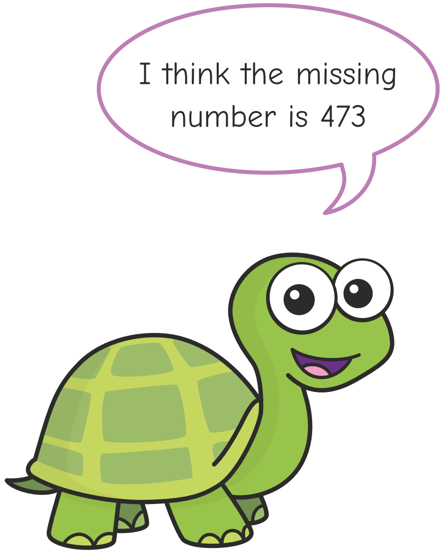
- Match the calculations that have the same results.

623 + 418	849 – 332
725 + 517	621 + 420
847 – 329	848 – 330
846 – 329	520 + 722

Find missing numbers

Reasoning and problem solving

327 + ☐ < 700



Explain the mistake Tiny has made.

Tiny has found the bond to 700 and ignored the inequality symbol.

48 + 37 > 38 + △

Give an example of what △ **could** be.

Give an example of what △ **could not** be.

What **must** be true about △ ?

any number less than 47

any number greater than or equal to 47

It must be less than 47

Write the missing digits to make the calculations correct.

__3__ + __3 = 300

__3__ – __3 = 300

How many possible solutions are there for each of the calculations?

237 + 63 = 300

333 – 33 = 300

Both calculations have only one possible solution.

Autumn Block 3

Multiplication and division A

Small steps

Step 1	Multiples
Step 2	Common multiples
Step 3	Factors
Step 4	Common factors
Step 5	Prime numbers
Step 6	Square numbers
Step 7	Cube numbers
Step 8	Multiply by 10, 100 and 1,000

Small steps

Step 9	Divide by 10, 100 and 1,000
Step 10	Multiples of 10, 100 and 1,000

Multiples

Notes and guidance

Children should already be familiar with the idea of multiples from their previous learning. They should understand that a multiple of a number is any number that is in its times-table. This can then be generalised to define a multiple more formally as the result of multiplying a number by a positive integer.

Building on this knowledge, children now find sets of multiples of given numbers and make generalisations about them. This allows children to begin to understand and use rules of divisibility, which will be built upon in later learning.

Children build multiples of numbers using concrete resources as well as pictorial representations. Arrays are particularly useful and will also help children when they study factors, prime numbers and square numbers later in the block. When listing multiples, children should work systematically to avoid omissions.

Things to look out for

- Children may confuse factors and multiples.
- Errors may be made with times-tables facts.
- Children may omit the number itself when listing multiples.
- Children may find it more difficult to identify and find multiples that go beyond the facts in the 12 times-table.

Key questions

- How do you find the multiples of a number?
- What do you notice about the multiples of ______?
  What is the same and what is different about them?
- Can a number be a multiple of more than one number?
- How can you tell if a number is a multiple of 2/5/10?
- What does the word “divisible” mean?
  How does it link to multiples?
- Are multiples of 8/4 also multiples of 4/8?

Possible sentence stems

- A multiple is the result of multiplying a number by ______
- The first multiple of a number is always ______
- ______ is a multiple of ______ because ______ × ______ = ______

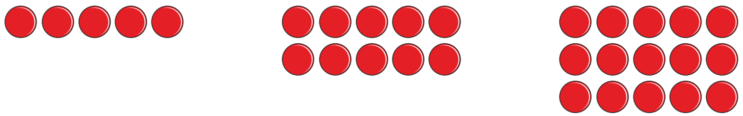
National Curriculum links

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Multiples

Key learning

- Here are the first three multiples of 5



Use counters to make these and the next three multiples of 5

List the first six multiples of 5

What is the same and what is different about the multiples of 5?

- How can you tell by looking at a number if it is a multiple of 5?

Which of these numbers are multiples of 5?

45	406	95	110	501	510	540	502

Which of the numbers are also multiples of 10?

- How can you tell by looking at a number if it is a multiple of 2?

Which of these numbers are multiples of 2?

62	48	104	401	86	68	612	620

Complete the sentence.

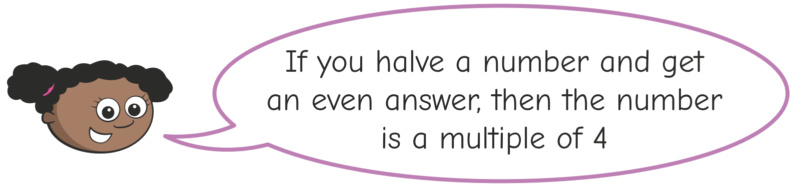
Multiples of 2 are called ______ numbers.

- List the first six multiples of 4

List the first six multiples of 8

What connection can you see between the multiples of 4 and the multiples of 8?

- Whitney has found a rule for identifying multiples of 4



Use Whitney's rule to find out which of the numbers are multiples of 4

64	46	104	82	90	96	120	248

Find a rule to test if a number is a multiple of 8

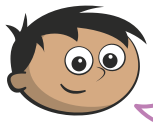
- On separate copies of a hundred square, shade all the multiples of each number.

2 3 4 5 6

What patterns do you notice?

Multiples

Reasoning and problem solving



If the sum of the digits of a number is a multiple of 3, then the number itself is a multiple of 3

Check Amir's rule using these multiples of 3

45	90	909	6,105

Use Amir's rule to find out whether these numbers are multiples of 3

81	103	267
1,524	5,810	

Amir's rule is correct.

81, 267 and 1,524 are multiples of 3

Find the sum of the digits of all the numbers in the 9 times-table up to 10 × 9

What do you notice?

Find the digit sums of these multiples of 9

648	8,388	9,378
82,602	99,999	

What do you notice?

What is the connection between numbers that are multiples of 9 and their digit sums?

The total is always 9

The total is a multiple of 9

Common multiples

Notes and guidance

Building on their knowledge from the previous step, children find common multiples of any pair of numbers. They do not need to be able to formally identify the lowest common multiple, but this idea can still be explored by considering the first common multiple of a pair of numbers.

Arrays and other representations may still be used for support, but children should start to become less reliant on these and more reliant on times-tables knowledge and simple rules of divisibility. These can be developed further as they notice, for example, that a multiple of 2 and 3 is also a multiple of 6 and can deduce that a number is divisible by 6 only if it is divisible by both 2 and 3

Encourage children to work systematically when listing multiples of given numbers. Tables and sorting diagrams are useful tools for children to show their results.

Things to look out for

- Children may confuse factors and multiples.
- Children may not be familiar with the use of the word "common" in this context.
- Children often think that the first common multiple of a pair of numbers is the product of the numbers.

Key questions

- How do you find the multiples of a number?
- What multiples do ______ and ______ have in common?
- What is the first multiple that ______ and ______ have in common?
- How can you tell if a number is a multiple of ______?
- Given any two numbers, can you always find a common multiple? How?

Possible sentence stems

- ______ is a multiple of ______ because ______ × ______ = ______
- ______ is a common multiple of ______ and ______ because ______ × ______ = ______ and ______ × ______ = ______
- The first common multiple of ______ and ______ is ______

National Curriculum links

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Common multiples

Key learning

- Here is a hundred square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade the first ten multiples of 5

Circle the first ten multiples of 3

List the first two common multiples of 5 and 3

What is the next common multiple of 5 and 3?

Find some more common multiples of 5 and 3

- On a hundred square, shade the first eight multiples of 6

Circle the first eight multiples of 4

List the first two common multiples of 6 and 4

Find some more common multiples of 6 and 4

- Nijah rings a bell every 6 seconds.

Dani blows a whistle every 8 seconds.

They start by ringing the bell and blowing the whistle at the same time.

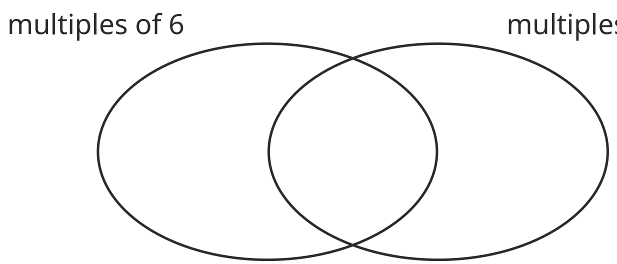
How many times will they ring the bell and blow the whistle at the same time in the next minute?

- Sort the numbers from 1 to 30 into the table.

	Multiple of 7	Not a multiple of 7
Multiple of 4		
Not a multiple of 4		

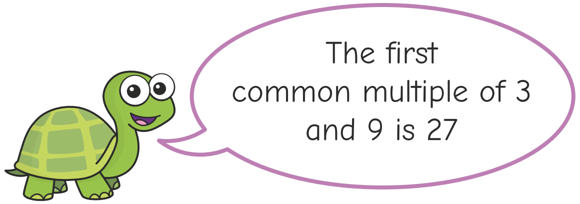
- Write the numbers in the sorting diagram.

12 | 18 | 24 | 9 | 6 | 45 | 48 | 54 | 36 | 63



Common multiples

Reasoning and problem solving



Tiny is wrong.

Find two numbers less than 27 that are multiples of both 3 and 9

9 and 18

Find different ways of completing the sentences.

All multiples of 10 are also multiples of ______ and ______

All multiples of 20 are also multiples of ______ and ______

All multiples of 30 are also multiples of ______ and ______

1, 2, 5

1, 2, 4, 5, 10

1, 2, 3, 5, 6, 10, 15

Are the statements always, sometimes or never true?

Common multiples of 2 and 3 are also multiples of 6

Common multiples of 5 and 10 are also multiples of 50

Explain your answers.

always true

sometimes true

Are the statements always, sometimes or never true?

The product of two even numbers is a multiple of an odd number.

The product of two odd numbers is a multiple of an even number.

Explain your answers.

always true

never true

Factors

Notes and guidance

Children explored the idea of factor pairs being multiplied together to produce a given number in Year 4. In this small step, they explore further the relationship between multiplication and division and consolidate their understanding of the words “factor” and “multiple”.

Children should know, for example, that as 5 is a factor of 20, 20 is a multiple of 5 and vice versa. They need to be aware of the special cases such as 1 being a factor of all numbers, and every number being both a multiple and a factor of itself. Children should also notice that although factors generally come in pairs, sometimes there is a repeated factor, for example 36 = 6 × 6, and this only needs to be listed once. This will be explored further later in the block.

Children begin to extend their knowledge by looking at products of three factors and products including simple multiples of powers of 10. Products using multiples of powers of 10 is looked at in depth in Step 10 of this block.

Things to look out for

- Children may confuse factors and multiples.
- Errors may be made with times-tables facts.
- Children may omit 1, the number itself or both when listing the factors of a number.

Key questions

- How do you find the factors of a number?
- How can you be sure you have found all the factors of a number?
- How can you work in a systematic way to find all the factors of a number?
- Do factors always come in pairs?
- Can a number be both a factor and a multiple of the same number?

Possible sentence stems

- ______ is a factor of ______ because ______ × ______ = ______
- ______ is a factor of ______ because ______ ÷ ______ = ______
- ______ is a factor of ______ because ______ is in the ______ times-table.

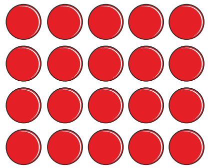
National Curriculum links

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Factors

Key learning

- The array shows that 4 and 5 are factors of 20



  How many other arrays can you make using 20 counters?

  Use your arrays to find all the factors of 20

- Which numbers are factors of 60?

9	6	8	4	12	5	60	15	45

  Which factors of 60 are not shown?

- Whitney has found the factors of 24

1 × 24	4 × 6
2 × 12	5 × ✗
3 × 8	

  Explain Whitney's method to a partner.

  How did she know when to stop?

  Use Whitney's method to find the factors of 42

-

40	75	57	35	505	705	507

  Which of the numbers is 5 a factor of? How do you know?

  Which of the numbers is 3 a factor of? How do you know?

-

40	80	82	66	56	106	160	144

  Which of the numbers is 2 a factor of?

  Which of the numbers is 4 a factor of?

  What do you notice?

- Complete the calculations.
  - 2 × ______ = 14 so 6 × 14 = 6 × 2 × ______
  - 3 × ______ = 9 so 9 × 12 = 3 × ______ × 12

- Scott knows that as 4 × 7 = 28, 4 × 70 = 280

  Complete the calculations.
  - ______ × 7 = 280
  - 4 × ______ = 2,800

Factors

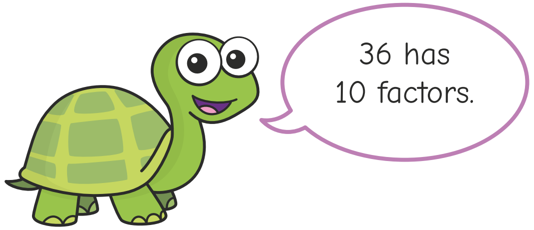
Reasoning and problem solving

Tiny has found the factors of 36

1	2	3	4	5	6
36	18	12	9	✗	6

Why does Tiny put a cross next to 5?

Why does Tiny stop after 6?



Do you agree with Tiny?

Explain your answer.

5 is not a factor of 36

Tiny would be repeating factors that have already been found.

No

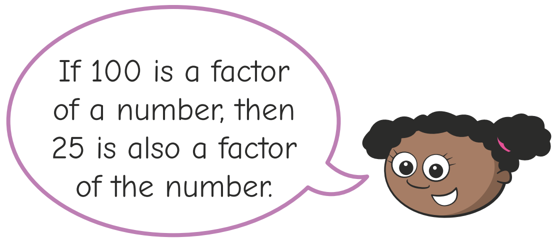
Are the statements always, sometimes or never true?

An even number has an even number of factors.

An odd number has an odd number of factors.

sometimes true

sometimes true



Is Whitney correct?

Can you replace 25 with another number?

Yes

You can replace 25 with any of 1, 2, 4, 5, 10, 20, 50

Common factors

Notes and guidance

In this small step, children learn that common factors are factors that are shared by two or more numbers.

Children work systematically to find lists of factors before comparing lists to find common factors. They should realise that 1 is a common factor of any set of numbers and that one of the numbers themselves could also sometimes be a common factor.

Arrays and other representations can be used as support when finding factors of numbers, including sorting diagrams for recording results. Children should use their times-tables knowledge as well as be able to recognise factors using the rules of divisibility.

Things to look out for

- Children may confuse factors and multiples.
- Children may not be familiar with the use of the word "common" in this context.
- Children may over-generalise the idea of pairs and think that a set of numbers can only have two common factors.
- It is common to omit 1 when listing factors, leading to an incorrect conclusion that a pair of numbers does not have a common factor.

Key questions

- Which numbers are factors of both the numbers?
- Which are the common factors of the numbers?
- On a sorting diagram, where can you see the common factors of the numbers?
- Why does any pair of numbers have at least one common factor?
- Can one of the numbers be a common factor? When does this happen?

Possible sentence stems

- ______ is a multiple of ______, so ______ is a factor of ______
- ______ is a factor of ______ and a factor of ______, so ______ is a common factor of ______ and ______
- The common factors of ______ and ______ are ______

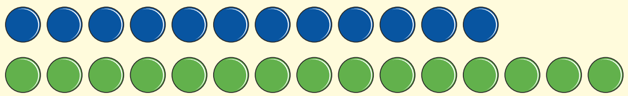
National Curriculum links

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

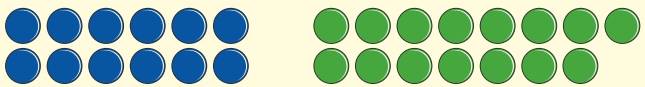
Common factors

Key learning

- Tiny is using arrays to find the common factors of 12 and 15



Both numbers can be arranged in one row, so 1 is a common factor.



12 can be arranged in two rows but 15 cannot, so 2 is not a common factor.

Working systematically, continue Tiny's method until you find all the common factors of 12 and 15

- List all the factors of 8

  List all the factors of 20

  What are the common factors of 8 and 20?

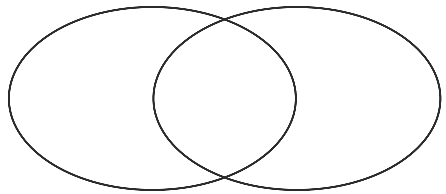
  How many common factors do 8 and 20 have?

- Write all the factors of 50 that are also factors of 25

- Write the numbers in the sorting diagram.

1	5	8	10	15	20

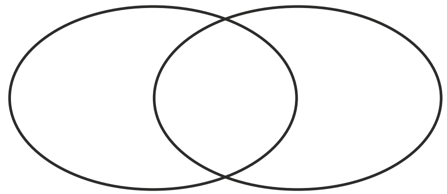
factors of 30 factors of 40



What other numbers can you add to the diagram?

- Complete the sorting diagram to show the factors of 20 and 24

factors of 20 factors of 24



What are the common factors of 20 and 24?

Use a sorting diagram to find the common factors of 9 and 15

- Find the common factors of each pair of numbers.

10 and 15	15 and 20	10 and 20

Common factors

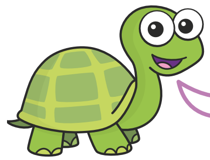
Reasoning and problem solving

Are the statements true or false?

1 is a multiple of every number.	False
0 is a factor of every number.	False
1 is a common factor of every pair of numbers.	True
2 is a common factor of every pair of even numbers.	True
5 is a common factor of every pair of multiples of 10	True
10 is a common factor of every pair of multiples of 5	False

Explain your answers.

Tiny is thinking of two numbers.



The common factors of my numbers are 1, 3, 7 and 21

What could Tiny's numbers be?

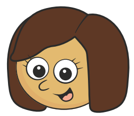
multiple possible answers, e.g.

21 and 42

21 and 63

42 and 63

Kim is thinking of two 2-digit numbers.



Both numbers have a digit sum of 6
Their common factors are 1, 2, 3, 4, 6 and 12

What are Kim's numbers?

24 and 60

Prime numbers

Notes and guidance

Building on their knowledge of factors, in this small step, children learn that numbers with exactly two factors are called prime numbers. They also learn that numbers with more than two factors are called composite numbers.

Through practice, children should recall the prime numbers up to 19. They should be able to determine whether numbers up to 100 are prime, using times-tables facts and the rules of divisibility they learned in earlier steps. Children use their knowledge of the concepts of both primes and factors to identify the prime factors of numbers. They learn that 1 is a special case as it is neither prime nor composite, as it has exactly one factor.

Things to look out for

- As most prime numbers are also odd numbers, children may mix up the two concepts and forget that 2 is a prime number.
- Children often mistake 1 for a prime number.
- Children may assume some numbers that do not appear in the times-tables up to 12 × 12 are prime, for example 51 = 3 × 17 is composite, not prime.
- Children may assume that all odd numbers are prime.

Key questions

- How many factors does the number have?
- How can you be sure you have found all the factors?
- What is the difference between a prime number and a composite number?
- How can you tell if a number is a multiple of 2/3/5?
- How can you check if a number is prime?
- How many factors does the number have?
  How many prime factors does the number have?

Possible sentence stems

- The only factors of ______ are ______ and ______, so ______ is prime.
- ______ is prime and a factor of ______, so ______ is a prime factor of ______

National Curriculum links

- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19

Prime numbers

Key learning

- All of these numbers are prime numbers.

5	13	17	23

  Use counters to find the factors of each number.

  What do you notice?

- A prime number has exactly two factors: 1 and itself.

  A composite number has more than two factors.

  Which of the numbers are prime and which are composite?

7	8	11	20	21	25	29

- On a hundred square, shade the number 1

  Shade the multiples of 2 apart from 2

  Shade the multiples of 3 apart from 3

  Continue this up to multiples of 7

  What numbers are you left with?

  What do you notice?

- Sort the numbers into the table.

  12 2 7 20 9 15 3 17 21

	Prime	Composite
Even		
Odd		

- List the factors of 20

  How many of the factors of 20 are prime?

- List the prime factors of 24

- Use your knowledge of multiples and factors to decide whether each number is prime.

70	92	51	61	81
29	57	43	63	77

Prime numbers

Reasoning and problem solving

Sort all the prime numbers between 10 and 100 into the table.

Number of ones			
1	3	7	9

Why do no 2-digit prime numbers have 0, 2, 4, 6 or 8 ones?

Why do no 2-digit prime numbers have 5 ones?

1: 11, 31, 41, 61, 71

3: 13, 23, 43, 53, 73, 83

7: 17, 37, 47, 67, 97

9: 19, 29, 59, 79, 89

They all have 2 as a factor.

They all have 5 as a factor.

Decide whether each statement is true or false.

All prime numbers are odd.

All odd numbers are prime.

The first prime number is 1

Talk about your answers with a partner.

False

False

False

Square numbers

Notes and guidance

In this small step, children use concrete manipulatives such as counters and cubes to build square numbers, and also to decide whether or not a given number is square. They learn that square numbers are the result of multiplying a number by itself. Through their knowledge of times-tables and practice over time, they should be able to recognise the square numbers up to 12 × 12. In this small step, children are introduced to notation for squared (^{2}).

Children explore the factors of square numbers and notice that they have an odd number of factors, because the number that multiplies by itself to make the square does not need a different factor to form a factor pair.

Things to look out for

- The notation for squared (^{2}) may confuse children, as they may think that $6^2 = 6 \times 2$ rather than 6×6
- Children may not realise that 1 is a square number, as its array may not appear to be a square.
- When listing factors, children may include the repeated factor twice, meaning they will not recognise that square numbers have an odd number of factors.

Key questions

- Why are square numbers called "square" numbers?
- How do you work out ______ squared?
- How do you write ______ squared?
- Is 1 a square number? Why or why not?
- Are the squares of odd numbers even or odd?
- Are the squares of even numbers even or odd?

Possible sentence stems

- A square number is the result of multiplying a number by ______
- ______ is a square number because ______ × ______ = ______
- ______ squared means ______ × ______ and is the square number ______

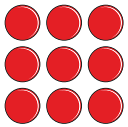
National Curriculum links

- Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Square numbers

Key learning

- 9 is a square number as 9 counters can be arranged to form a square array.

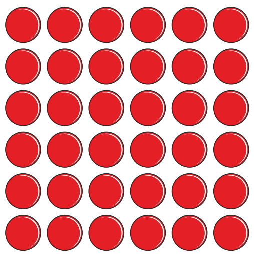


$3 \times 3 = 9$

Use counters to decide whether each number is square.

12	16	4	10	20	25	24

- 36 counters can be arranged into a square array with 6 rows and 6 columns.



$6 \times 6 = 36$

How many counters will there be in a square array with 7 rows and 7 columns?

How many counters will there be in a square array with 8 rows and 8 columns?

How many counters will there be in a square array with 10 rows and 10 columns?

- A square number is found by multiplying a number by itself.

$5^2 = 5 \times 5$ and is said as "5 squared".

What is the value of 5^2?

Work out the values of the square numbers.

4^2	9^2	10^2	8^2	7^2

- Esther thinks $6^2 = 12$

Do you agree?

Explain your answer.

- Here are five digit cards.

1	2	5	6	7

Choose two cards each time to make:

- an even number
- a square number
- a prime number
- a multiple of 9
- a factor of 48
- an even square number

Square numbers

Reasoning and problem solving

List the first six square numbers and find their factors.

How many factors does each square number have?

What do you notice about the number of factors that square numbers have?

Explain why this happens.

Each square number has an odd number of factors.

Some square numbers can be written as the sum of two prime numbers.

Here is an example. 2 + 2 = 4

Find some other square numbers that can be written as the sum of two prime numbers.

multiple possible answers, e.g.

2 + 7 = 9

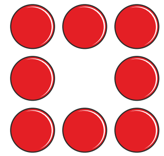
5 + 11 = 16

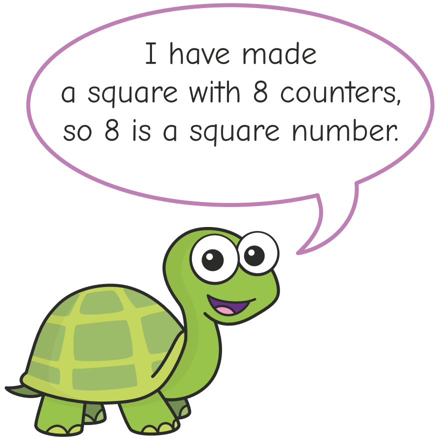
23 + 2 = 25

29 + 7 = 36

47 + 2 = 49

Tiny is using counters to make square numbers.





Do you agree with Tiny?

Explain your answer.

No

Cube numbers

Notes and guidance

In this small step, children learn that a cube number is the result of multiplying a whole number by itself and then by itself again, for example 6 × 6 × 6. Linking this to previous learning on square numbers, children should recognise that when they multiply a number by itself once, the result is a square number, and so to find the cube of a given number, they can multiply its square by the number itself, for example 6 × 6 = 36, so 6 cubed = 36 × 6. Children are introduced to the notation for cubed (^{3}) for the first time and should ensure that this is not confused with the notation for squared (^{2}) from the previous step.

Cube numbers could be introduced through using interlocking cubes to make larger cubes. This can be related to finding the volume of cubes and cuboids, which is introduced in the Summer term and studied more formally in Year 6

Things to look out for

- The notation for cubed (^{3}) may confuse children, as they may think that $6^3 = 6 \times 3$ rather than 6 × 6 × 6
- Children may not realise that 1 is a cube number.
- Children may think that to find the cube of a number they can square it and then square the result.

Key questions

- Why are cube numbers called "cube" numbers?
- How do you work out ______ cubed?
- How do you write ______ cubed?
- Is 1 a cube number? Explain your answer.
- Are the cubes of odd numbers even or odd?
- Are the cubes of even numbers even or odd?

Possible sentence stems

- The cube of a number is the result of multiplying the number by ______ and then by ______ again.
- ______ is a cube number because ______ × ______ × ______ = ______
- ______ cubed means ______ × ______ × ______ and is the cube number ______

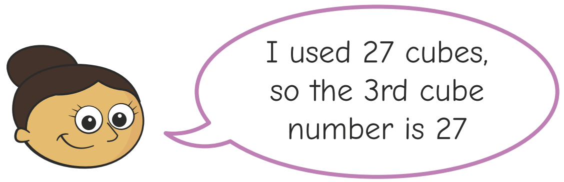
National Curriculum links

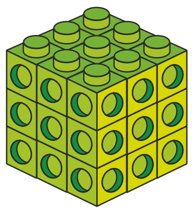
- Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes

Cube numbers

Key learning

- Dora has used small cubes to make a cube with a side length of 3





Use cubes to work out the 2nd cube number.

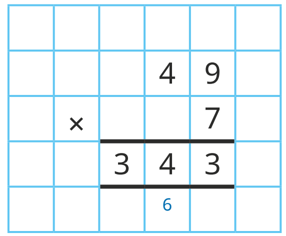
- Complete the table.

Size of cube	Calculation	Number of cubes
1^3		1
2^3		8
3^3	$3 \times 3 \times 3$	
4^3		
5^3		
6^3	$6 \times 6 \times 6$	

- Filip is using square numbers to help work out cube numbers.

Here are his workings.

$7^3 = 7 \times 7 \times 7$
$= 49 \times 7$
$= 343$



Use Filip's method to work out 8^3 and 9^3

- Write <, > or = to compare the calculations.

5 squared ◯ 4 cubed

5^3 ◯ 8^2

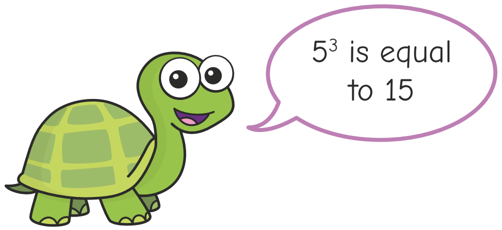
1^2 ◯ 1^3

45 squared ◯ 45 cubed

- Show that the sum of 3^3 and 7^3 is **not** equal to 10^3

Cube numbers

Reasoning and problem solving



Do you agree with Tiny?

Explain your answer.

No

Here are three cards.

A	B	C

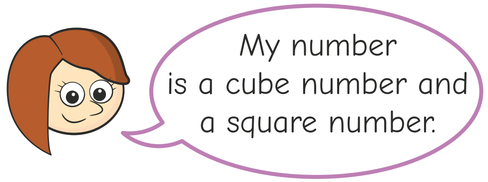
Each card represents a cube number.

Use the clues to work out the numbers.

- A × A = B
- B + B – 3 = C
- digit sum of C = A

A = 8, B = 64,
C = 125

Rosie is thinking of a number less than 100

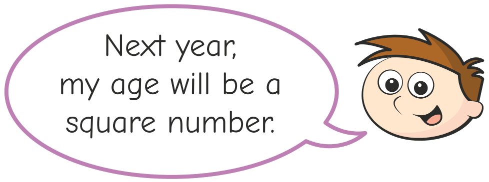


What number could Rosie be thinking of?

Is there more than one possible answer?

1 or 64

Teddy's age is a cube number.



How old is Teddy now?

8 years old

Multiply by 10, 100 and 1,000

Notes and guidance

In this small step, children revisit multiplying whole numbers by 10 and 100 (introduced in Year 4), and move on to multiplying whole numbers by 1,000

Concrete manipulatives such as place value charts and counters and Gattegno charts can be used to support understanding, using children's knowledge of the relationship between digits in given rows/columns.

Children need to be aware that the effect of multiplying by 10 twice is the same as multiplying by 100 and that multiplying by 10 three times is the same as multiplying by 1,000. Children should be comfortable with the language of "10 times the size of", "100 times the size of" and "1,000 times the size of".

In the next steps, children look at dividing whole numbers by 10, 100 and 1,000 and then multiplying and dividing by multiples of 10, 100 and 1,000

Things to look out for

- Children may move digits in the wrong direction in the place value chart, or by the wrong number of columns.
- Some children may over-generalise that multiplying by a power of 10 always results in adding zeros, which will cause issues in the Spring term when multiplying decimals.

Key questions

- In what direction do the digits move when you multiply by 10/100/1,000?
- How many places to the left do the digits move when you multiply by 10/100/1,000?
- When you have an empty place value column, what digit do you use as a placeholder?
- How can you use the result of multiplying by 100 to help you multiply a number by 1,000?

Possible sentence stems

- ______ multiplied by 10/100/1,000 is equal to ______
- ______ is 10/100/1,000 times the size of ______
- There were ______ ones/tens. Now there are ______ tens/hundreds.
- Multiplying by 100 is the same as multiplying by ______ twice.

National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

Multiply by 10, 100 and 1,000

Key learning

- Use counters to make 234 on a place value chart.

HTh	TTh	Th	H	T	O
			●●	●●●	●●●●

If you multiply 234 by 10, where do the counters move to?

What is the result of multiplying 234 by 10?

If you multiply 234 by 100, where do the counters move to?

What is the result of multiplying 234 by 100?

- Complete the calculations.

You can use a place value chart to help you.

  - 156 × 100 = ______
  - 100 × 36 = ______
  - 45,020 × 10 = ______
  - ______ = 324 × 100
  - 1,000 × 207 = ______
  - ______ = 3,406 × 100

- Work out the calculations.

37 × 10	37 × 100	37 × 1,000

What is the same and what is different?

- Complete the multiplications.

  - 4 × 10 = ______
    4 × 100 = ______
    4 × 1,000 = ______
  - 24 × 10 = ______
    24 × 100 = ______
    24 × 1,000 = ______
  - 204 × 10 = ______
    204 × 100 = ______
    204 × 1,000 = ______
  - 240 × 10 = ______
    240 × 100 = ______
    240 × 1,000 = ______

What do you notice?

- Write <, > or = to complete the statements.

71 × 1,000 ◯ 71 × 100

100 × 32 ◯ 16 × 1,000

6×10^3 ◯ 45×10^2

- What number is 100 times the size of 4,000?

4,000 is 100 times the size of what number?

Multiply by 10, 100 and 1,000

Reasoning and problem solving

100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

What number is 1,000 times the size of the number shown?

Show 602,000 on a Gattegno chart.

Use the chart to work out the missing numbers.

602,000 = ______ × 10

602,000 = ______ × 100

602,000 = ______ × 1,000

463,000

60,200, 6,020, 602

Aisha has won 300 points in a computer game.

Brett has 100 times the number of points Aisha has.

How many more points does Brett have than Aisha?

29,700

Ms Rose has £1,020

Mr Trent has £120

Is Tiny correct?

Explain your reasoning.

No

Divide by 10, 100 and 1,000

Notes and guidance

In this small step, children revisit dividing numbers by 10 and 100, and move on to dividing whole numbers by 1,000

As with multiplying, place value charts, counters and Gattegno charts can be used to support understanding, using children's knowledge of relationships between rows and columns. They need to be aware that the effect of dividing by 10 twice is the same as dividing by 100 and that dividing by 10 three times is the same as dividing by 1,000. Children should be comfortable with the language of "one-tenth the size of", "one-hundredth the size of" and "one-thousandth the size of".

Children should be aware that multiplication and division are inverse operations and make links between this step and previous learning.

Division with decimal answers is covered in the Spring term.

Things to look out for

- Children may move digits in the wrong direction in the place value chart, or by the wrong number of columns.
- Children may make errors with the number of zeros at the end of a number and/or zeros used as placeholders.

Key questions

- What direction do the digits move when you divide by 10/100/1,000?
- How many places to the right do digits move when you divide by 10/100/1,000?
- How is dividing by 10, 100 or 1,000 linked to multiplying by 10, 100 or 1,000?
- How can you use the result of dividing by 100 to help you divide a number by 1,000?
- What does "inverse" mean?

Possible sentence stems

- _______ divided by 10/100/1,000 is equal to _______
- _______ is one-tenth/one-hundredth/one-thousandth the size of _______
- There were _______ tens/hundreds. Now there are _______ ones/tens.

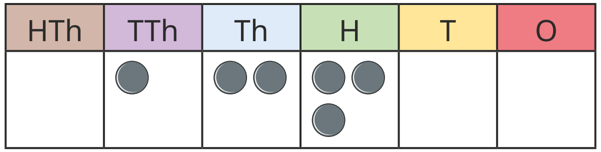
National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

Divide by 10, 100 and 1,000

Key learning

- What number is represented in the place value chart?



If you divide the number by 10, where do the counters move to?

What is the result of dividing the number by 10?

If you divide the number by 100, where do the counters move to?

What is the result of dividing the number by 100?

- Use a place value chart or a Gattegno chart to work out the calculations.

460 ÷ 10	5,300 ÷ 100	62,000 ÷ 1,000

- Divide each number by 10, 100 and 1,000

80,000	300,000	547,000

- Work out 45,000 ÷ 10 ÷ 10

  How else could you write this calculation?

  How else could you write 45,000 ÷ 10 ÷ 10 ÷ 10?

- Complete the divisions.

  - 64,000 ÷ 10 = ______

    64,000 ÷ 100 = ______

    64,000 ÷ 1,000 = ______

HTh	TTh	Th	H	T	O
	6	4	0	0	0

  - 604,000 ÷ 10 = ______

    604,000 ÷ 100 = ______

    604,000 ÷ 1,000 = ______

HTh	TTh	Th	H	T	O
6	0	4	0	0	0

- Complete the calculations.

  - 180,000 ÷ ______ = 180
  - ______ ÷ 1,000 = 22
  - ______ × 1,000 = 66,000
  - ______ × 100 = 4,000
  - 180,000 ÷ ______ = 18,000
  - ______ ÷ 100 = 402
  - ______ × 100 = 9,700
  - ______ ÷ 100 = 4,000

Divide by 10, 100 and 1,000

Reasoning and problem solving

Mr Xu has £357,000 of savings in his bank account.

He takes one-thousandth of his savings out of his bank account.

Using this money, he buys a suit costing £269

How much of the money that he took out of the bank does Mr Xu have left?

£88

Complete the calculations.

______ × 100 = 600

______ + 100 = 600

______ ÷ 1,000 = 600

______ – 1,000 = 600

6

500

600,000

1,600

Is the statement always, sometimes or never true?

Dividing by 100 is the same as dividing by 10 twice.

Explain your answer.

always true

Ron has 400 marbles.

Jack has ten times as many marbles as Ron.

Eva has one-tenth of the number of marbles that Ron has.

How many marbles do Ron, Jack and Eva have altogether?

4,440

Multiples of 10, 100 and 1,000

Notes and guidance

In this small step, children build on previous learning and begin to multiply and divide by multiples of 10, 100 and 1,000.

Children use knowledge of factors to break a calculation down into a series of easier calculations. For example, to multiply by 200, they write 200 as 2 × 100 and then multiply by 2 and by 100. Children use the commutative law to know that they can find the product by multiplying by the factors in either order.

Children use their knowledge of multiples and factors of numbers in common times-tables and link this to powers of 10 to find multiples of related numbers. They also work out related multiplications and divisions from a given fact that uses multiples of powers of 10

Things to look out for

- Children may mix up the operations they need to use, for example mistakenly thinking that because 400 = 100 × 4, dividing by 400 is the same as dividing by 100 and then multiplying by 4
- At first, children may need support to recognise the relationships between calculations such as 36 × 5 and 36 × 50

Key questions

- Will multiplying/dividing by 20 give an answer that is less than or greater than multiplying/dividing by 10? Why?
- How can you break down multiplying/dividing by ______ into steps using powers of 10?
- What is the same and what is different about the two calculations?
- How can you use inverse operations to find related calculations?
- When do numbers have common multiples that are lower than their product?

Possible sentence stems

- ______ = ______ × ______, so to multiply by ______ you can first multiply by ______ and then by ______
- ______ = ______ × ______, so to divide by ______ you can first divide by ______ and then by ______

National Curriculum links

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- Multiply and divide numbers mentally, drawing upon known facts

Multiples of 10, 100 and 1,000

Key learning

- Here are two methods to work out 24 × 20

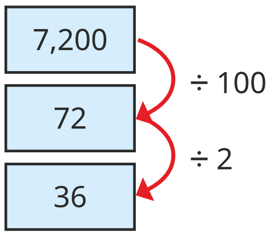
Method 1	Method 2
24 × 10 × 2 = 240 × 2 = 480	24 × 2 × 10 = 48 × 10 = 480

  What is the same and what is different about the two methods?

  Work out the multiplications.

21 × 30	43 × 20	12 × 400	2,000 × 13

- The diagram shows that 7,200 ÷ 200 = 36



  Use a similar strategy to work out the divisions.

18,000 ÷ 200	3,600 ÷ 300	3,600 ÷ 30	8,800 ÷ 400

- Work out the multiplications.

  Show all the steps in your thinking.

6 × 400	60 × 400	30 × 8,000	400 × 500

- Find a number for each clue.
  - a multiple of 30 that is between 100 and 200
  - a multiple of 40 that is between 300 and 400
  - a multiple of 500 that is between 4,000 and 5,000

- Use the fact that 36 × 5 = 180 to find the answers to the calculations.

36 × 50	5 × 360	180 ÷ 5
500 × 36	360 × 500	1,800 ÷ 5

- Teddy has 8 boxes of 50 apples.

  Rosie has 5 boxes of 80 apples.

  How many apples do they each have?

  What do you notice? Why does this happen?

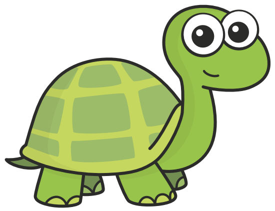
Multiples of 10, 100 and 1,000

Reasoning and problem solving

Tiny is working out 600 ÷ 25

Here are Tiny's workings.

600 ÷ 25
600 ÷ 2 = 300
300 ÷ 5 = 60
600 ÷ 25 = 60

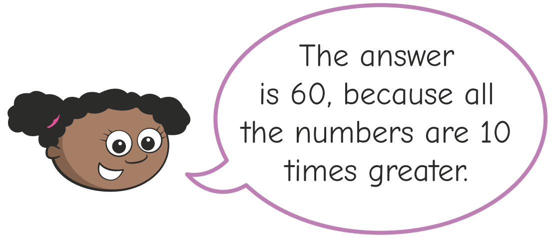


Explain why Tiny is incorrect.

Find the correct answer.

24

Whitney is using the fact that 6 × 7 = 42 to work out 420 ÷ 70



Do you agree with Whitney?

Explain your answer.

No

Which is the correct way to work out 800 ÷ 25?

A	B
800 ÷ 100 = 8 8 ÷ 4 = 2	800 ÷ 100 = 8 8 × 4 = 32

Explain your answer.

B

Autumn Block 4

Fractions A

Small steps

Step 1	Find fractions equivalent to a unit fraction
Step 2	Find fractions equivalent to a non-unit fraction
Step 3	Recognise equivalent fractions
Step 4	Convert improper fractions to mixed numbers
Step 5	Convert mixed numbers to improper fractions
Step 6	Compare fractions less than 1
Step 7	Order fractions less than 1
Step 8	Compare and order fractions greater than 1

Small steps

Step 9	Add and subtract fractions with the same denominator
Step 10	Add fractions within 1
Step 11	Add fractions with total greater than 1
Step 12	Add to a mixed number
Step 13	Add two mixed numbers
Step 14	Subtract fractions
Step 15	Subtract from a mixed number
Step 16	Subtract from a mixed number – breaking the whole

Small steps

Step 17	Subtract two mixed numbers

Find fractions equivalent to a unit fraction

Notes and guidance

Children are familiar with the idea of equivalent fractions from earlier study. This small step focuses on how unit fractions can be expressed in other forms.

It is important that children use a variety of representations, including fractions of shapes, number lines and fraction walls as well as the abstract form, so that they understand the relationships. They complement this conceptual understanding by using their times-table knowledge to find missing numerators or denominators, working both horizontally and vertically.

Children move on to find fractions equivalent to non-unit fractions in the next step and use this learning throughout the block.

Things to look out for

- Children may not understand that different fractions that represent the same amount are equivalent, for example $\frac{1}{3}$ and $\frac{2}{6}$
- Children may over-generalise to "do the same to the numerator and denominator" and use incorrect additive instead of multiplicative relationships, for example $\frac{1}{4} = \frac{2}{5}$ because 1 + 1 = 2 and 4 + 1 = 5

Key questions

- What does "equivalent" mean?
- What is a unit fraction?
- When are two fractions equivalent?
- How can you use the model to see if the two fractions are equivalent?
- How do you use a fraction wall to find equivalent fractions?
- What multiplication/division facts can you use?

Possible sentence stems

- A fraction is a unit fraction if the ______ is equal to ______
- The numerator has been multiplied/divided by ______, so if the denominator is multiplied/divided by ______, then the fractions will be equivalent.
- The denominator is ______ times the numerator in both fractions, so the fractions are ______

National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Find fractions equivalent to a unit fraction

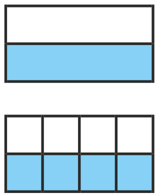
Key learning

- Take two pieces of paper that are the same size.

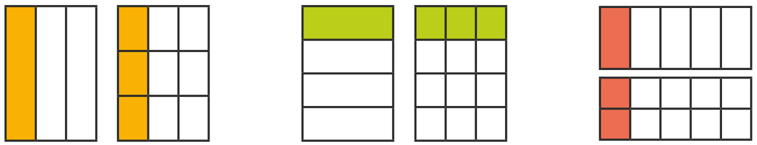
  Fold one piece into 2 equal parts and the other piece into 8 equal parts.

  Explain how the pieces of paper show that $\frac{1}{2} = \frac{4}{8}$

  Use more pieces of paper to find other fractions equivalent to one half.

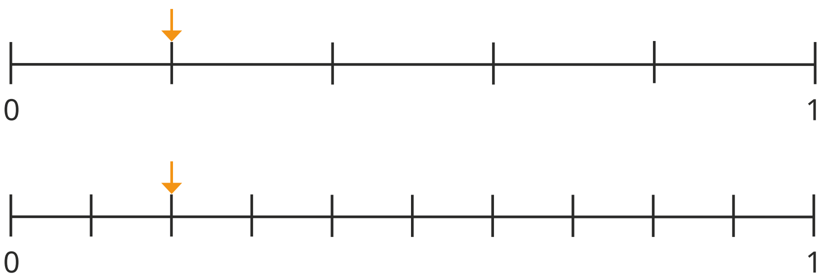


- Use the models to write equivalent fractions.

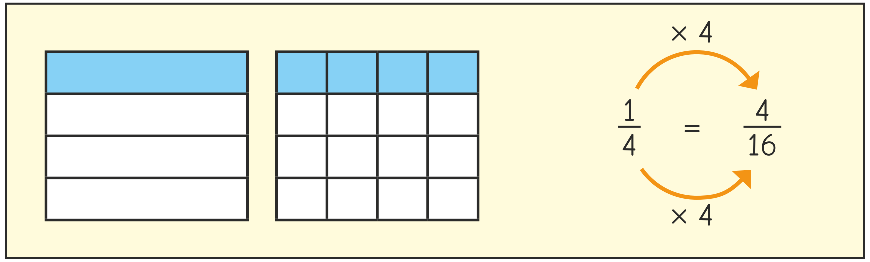


  Make or draw more models to show other fractions equivalent to unit fractions.

- How do the number lines show that $\frac{1}{5}$ is equivalent to $\frac{2}{10}$?



- Whitney uses diagrams and multiplication and division skills to find equivalent fractions.



  Use Whitney's method to find the missing numbers.

  - $\frac{1}{2} = \frac{\square}{12}$
  - $\frac{1}{5} = \frac{\square}{30}$
  - $\frac{4}{\square} = \frac{1}{3}$
  - $\frac{\square}{6} = \frac{5}{30}$

- Amir uses the relationships between the numerators and denominators to find equivalent fractions.

  × 7 $\frac{1}{7} = \frac{4}{28}$ × 7 ÷ 5 $\frac{3}{15} = \frac{1}{5}$ ÷ 5

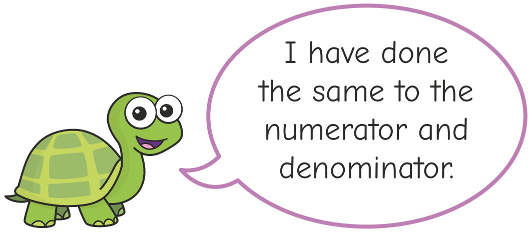
  Use Amir's method to find the missing numbers.

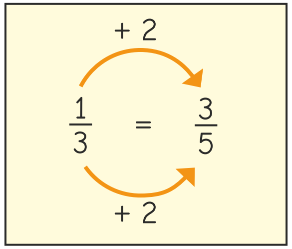
  - $\frac{1}{2} = \frac{8}{\square}$
  - $\frac{1}{5} = \frac{\square}{20}$
  - $\frac{1}{12} = \frac{\square}{36}$
  - $\frac{\square}{3} = \frac{10}{30}$

Find fractions equivalent to a unit fraction

Reasoning and problem solving

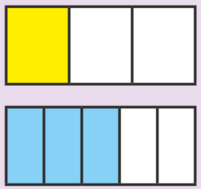
Tiny is finding fractions that are equivalent to one third.





Use a diagram to show that Tiny is wrong.

How many sixths are equivalent to one third?



$\frac{1}{3} = \frac{2}{6}$

Are the statements true or false?

$\frac{1}{2} = \frac{10}{20}$	$\frac{1}{3} = \frac{15}{30}$
$\frac{1}{4} = \frac{40}{400}$	$\frac{1}{5} = \frac{20}{100}$
$\frac{1}{6} = \frac{12}{66}$	$\frac{1}{7} = \frac{4}{10}$

Explain your answers.

True False

False True

False False

Complete the set of equivalent fractions.

$\frac{1}{6} = \frac{\square}{12} = \frac{\square}{18} = \frac{4}{\square} = \frac{\square}{30} = \frac{6}{\square} = \frac{7}{\square}$

2, 3, 24, 5, 36, 42

Find fractions equivalent to a non-unit fraction

Notes and guidance

Building from the previous step, in this small step children find fractions that are equivalent to a non-unit fraction.

Children continue to use a variety of representations, including fractions of shapes, number lines and parts of a fraction wall as well as the abstract form, to understand the relationships. They complement this conceptual understanding by using multiplication and division facts to find missing numerators or denominators when working in the abstract.

The understanding gained in this and the previous step will help children to recognise equivalent fractions in the next step and prepare them for when they add and subtract fractions with different denominators later in the block.

Things to look out for

- Children may not understand that different fractions that represent the same amount are equivalent, for example $\frac{2}{3}$ and $\frac{4}{6}$
- Children may over-generalise to "do the same to the numerator and denominator" and use incorrect additive instead of multiplicative relationships, for example $\frac{3}{4} = \frac{4}{5}$ because 3 + 1 = 4 and 4 + 1 = 5

Key questions

- What does "equivalent" mean?
- When are two fractions equivalent?
- How can you use the diagram to see if the two fractions are equivalent?
- How can you use your knowledge about unit fractions to help with non-unit fractions?
- How do you use a fraction wall to find equivalent fractions?
- What multiplication/division facts can you use?

Possible sentence stems

- The numerator has been multiplied/divided by ______, so if the denominator is multiplied/divided by ______, then the fractions will be equivalent.
- I know that ______ is equivalent to ______ because …

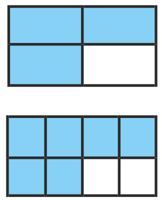
National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Find fractions equivalent to a non-unit fraction

Key learning

- Take two pieces of paper that are the same size.

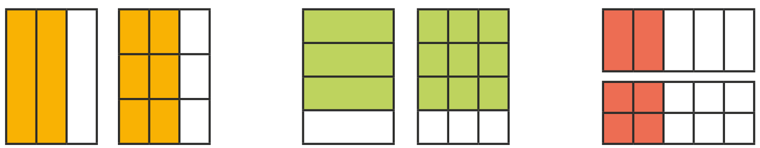
  

  Fold one piece into 4 equal parts and the other piece into 8 equal parts.

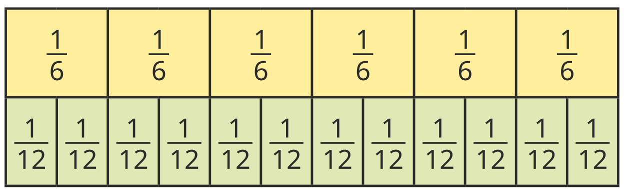
  Explain how the pieces of paper show $\frac{3}{4} = \frac{6}{8}$

  Use more pieces of paper to find other fractions equivalent to $\frac{3}{4}$

- Use the diagrams to write equivalent fractions.

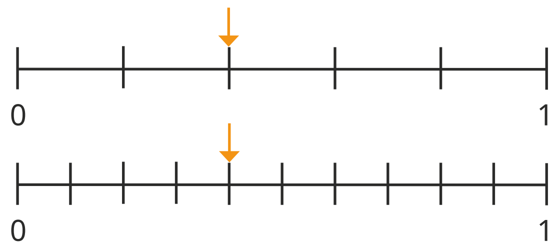
  Make or draw more diagrams to show other fractions equivalent to non-unit fractions.

- 

  Use the bar model to complete the equivalent fractions.

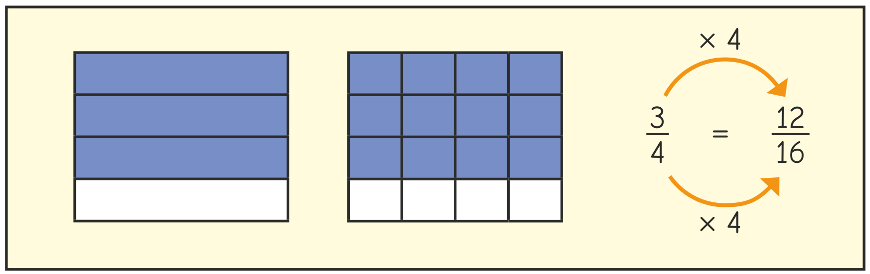
  - $\frac{2}{6} = \frac{\square}{12}$
  - $\frac{3}{6} = \frac{\square}{12}$
  - $\frac{4}{6} = \frac{\square}{12}$
  - $\frac{5}{6} = \frac{\square}{12}$
  - $\frac{6}{6} = \frac{\square}{12}$

- How do the number lines show that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$?

  What other equivalent fractions can be seen from the number lines?

- Scott uses diagrams and multiplication and division skills to find equivalent fractions.

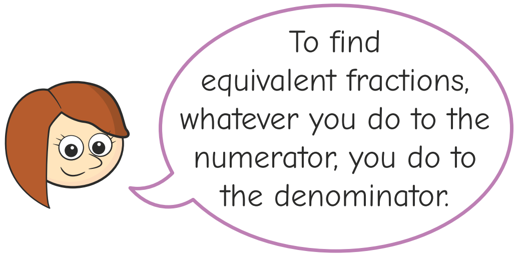
  Use Scott's method to find the missing numbers.

  - $\frac{3}{5} = \frac{\square}{20}$
  - $\frac{4}{9} = \frac{\square}{27}$
  - $\frac{16}{\square} = \frac{2}{3}$
  - $\frac{\square}{6} = \frac{25}{30}$

Find fractions equivalent to a non-unit fraction

Reasoning and problem solving

Rosie is working out some equivalent fractions.



A: $\frac{4}{8} = \frac{8}{16}$

B: $\frac{4}{8} = \frac{6}{10}$

C: $\frac{4}{8} = \frac{2}{4}$

D: $\frac{4}{8} = \frac{1}{5}$

Which of Rosie's equivalent fractions are correct?

For any incorrect answers, explain the mistake Rosie has made.

A and C

Here are some fraction cards.

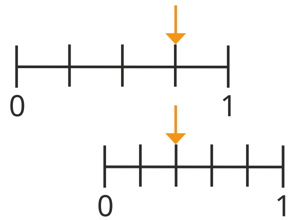
$\frac{4}{A}$ $\frac{B}{C}$ $\frac{20}{50}$

Use the clues to work out the values of A, B and C.

- All three fractions are equivalent.
- A + B = 16

A = 10, B = 6,
C = 15

Tiny thinks that the number lines show that $\frac{3}{4}$ is equivalent to $\frac{2}{5}$



Is Tiny correct?

Explain your answer.

No

Recognise equivalent fractions

Notes and guidance

Children develop their learning from the previous two steps to recognise pairs and larger sets of equivalent fractions.

Various methods are explored, including looking for common factors and multiples to establish whether fractions are equivalent, and also looking at the multiplicative relationship between the numerator and denominator. The use of diagrams and other pictorial representations are used throughout to support children's understanding of the abstract methods.

The key point of this step is to recognise equivalent fractions, and although this includes some simplifying, there is no need to focus on writing fractions in their simplest form, which is covered in Year 6

Things to look out for

- Errors may occur in finding the common factors of the numerator and denominator.
- Children may get confused when looking for the relationships between numerators and denominators.
- Children may over-generalise to "do the same to the numerator and denominator" and use incorrect additive instead of multiplicative relationships, for example $\frac{3}{4} = \frac{4}{5}$ because 3 + 1 = 4 and 4 + 1 = 5

Key questions

- What does "equivalent" mean?
- When are two fractions equivalent?
- How can you use a fraction wall to check if the fractions are equivalent?
- What are the common factors of the numerator and the denominator?
- Are there any other factors you could use?
- What is the relationship between the numerator and the denominator of the fractions?

Possible sentence stems

- ______ is a common factor of the numerator and the denominator, so I can divide both of these by ______ to find an equivalent fraction.
- The numerator/denominator has been multiplied by ______, so the denominator/numerator should also be ______ by ______

National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

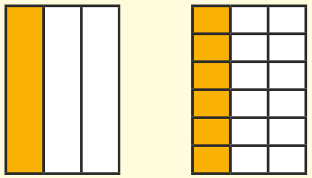
Recognise equivalent fractions

Key learning

- Alex has shown that $\frac{1}{3}$ is equivalent to $\frac{6}{18}$

$$\frac{1}{3} \overset{\times 6}{=} \frac{6}{18}$$

× 6, × 3, × 3, × 6



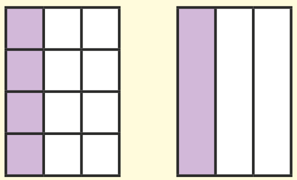
Show that the fractions are equivalent.

$\frac{1}{3} = \frac{5}{15}$ $\frac{1}{5} = \frac{4}{20}$ $\frac{1}{2} = \frac{8}{16}$

- Mo has shown that $\frac{4}{12}$ is equivalent to $\frac{1}{3}$

$$\frac{4}{12} \overset{\div 4}{=} \frac{1}{3}$$

÷ 4, ÷ 4



Show that the fractions are equivalent.

$\frac{12}{15} = \frac{4}{5}$ $\frac{16}{28} = \frac{4}{7}$ $\frac{10}{12} = \frac{5}{6}$ $\frac{12}{30} = \frac{2}{5}$

- Use multiplication or division facts alongside diagrams to decide which of the fractions are equivalent to $\frac{1}{5}$

$\frac{3}{15}$ $\frac{10}{50}$ $\frac{4}{25}$ $\frac{6}{30}$ $\frac{8}{45}$

- Use multiplication or division facts alongside diagrams to decide which of the fractions are equivalent to $\frac{3}{8}$

$\frac{6}{16}$ $\frac{30}{48}$ $\frac{15}{40}$ $\frac{12}{32}$ $\frac{9}{24}$

- Here are six fractions.

$\frac{1}{3}$ $\frac{9}{12}$ $\frac{12}{18}$ $\frac{3}{4}$ $\frac{4}{12}$ $\frac{2}{3}$

Sort the fractions into three pairs of equivalent fractions.

- Use the number cards to complete the equivalent fractions.

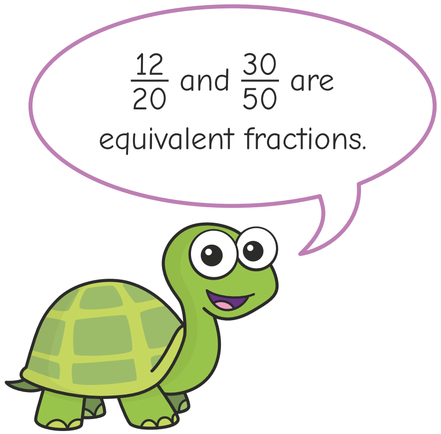
2   **15**   **10**   **3**

$$\frac{4}{6} = \frac{\square}{\square} = \frac{\square}{\square}$$

Recognise equivalent fractions

Reasoning and problem solving

Tiny is working out equivalent fractions.



Show that Tiny is correct.

Both fractions are equivalent to $\frac{3}{5}$

Are the fractions equivalent?

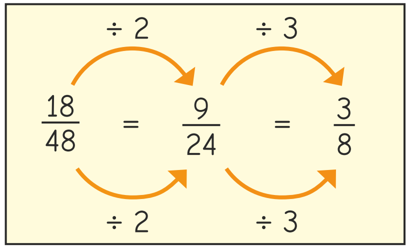
$\frac{30}{40}$ $\frac{36}{48}$

Explain your answer.

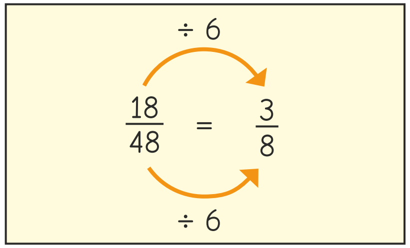
Yes

Sam and Eva find out that $\frac{18}{48}$ is equivalent to $\frac{3}{8}$

Here are Sam's workings.



Here are Eva's workings.



Whose method is more efficient?

Explain your answer.

Show that $\frac{60}{144}$ is equivalent to $\frac{5}{12}$

Eva's

multiple possible answers

Convert improper fractions to mixed numbers

Notes and guidance

Children encountered fractions greater than 1 and mixed numbers in Year 4

They may need reminding that an improper fraction is one where the numerator is greater than or equal to the denominator and a mixed number consists of an integer and a proper fraction.

Children use objects and diagrams to make wholes to support converting improper fractions into mixed numbers. Once they are confident with this as a concept, they move on to a more abstract approach using division and remainders. Understanding the whole is key to their understanding.

This skill is important for adding fractions and adding mixed numbers later in the block.

Things to look out for

- Children may not make connections between the denominator of a fraction and the number of equal parts needed to make one whole.
- Children may have the misconception that a fraction is always "part of a whole" and not realise that an improper fraction is a fraction that is greater than or equal to 1

Key questions

- How many ______ are there in one whole?
- How many ______ are there in 2/3/4 wholes?
- What does each part of a mixed number represent?
- What is an improper fraction?
- How many cubes do you need to represent the improper fraction? How can you use the cubes to make wholes? What do the remaining cubes represent?

Possible sentence stems

- There are ______ in one whole, so there are ______ in ______ wholes.
- I can regroup ______ to make ______ wholes with ______ parts left over. As a mixed number, this is ______ and ______

National Curriculum links

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Convert improper fractions to mixed numbers

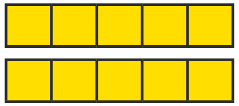
Key learning

- The bar model shows that $1 = \frac{6}{6}$

Work out the missing numbers.

▶ $1 = \frac{5}{\square}$ ▶ $1 = \frac{7}{\square}$ ▶ $1 = \frac{\square}{8}$ ▶ $\frac{\square}{4} = 1$

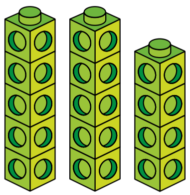
- The bar models shows that $2 = \frac{10}{5}$



Work out the missing numbers.

▶ $3 = \frac{\square}{5}$ ▶ $2 = \frac{\square}{4}$ ▶ ______ $= \frac{16}{4}$ ▶ $\frac{9}{3} =$ ______

- Nijah converts $\frac{14}{5}$ to a mixed number using cubes.



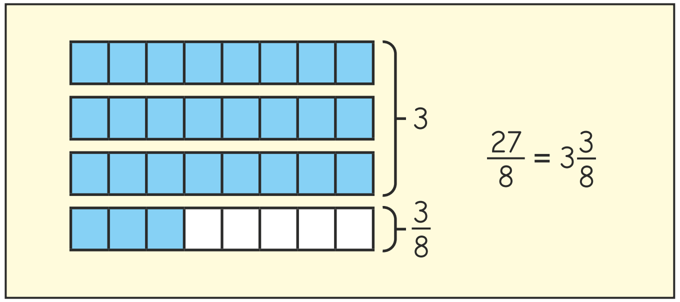
▶ Why has Nijah built towers of 5 cubes?

▶ How many full towers of 5 has Nijah built?

▶ How many cubes are in the last tower?

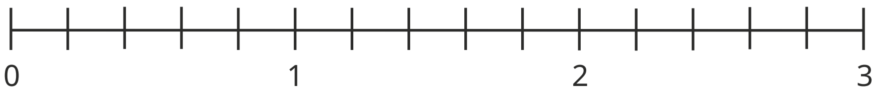
▶ Write $\frac{14}{5}$ as a mixed number.

- Tommy uses a bar model to convert the improper fraction $\frac{27}{8}$ to a mixed number.



Use Tommy's method to convert $\frac{25}{8}$, $\frac{26}{8}$, $\frac{18}{7}$ and $\frac{19}{4}$ to mixed numbers.

- Label $\frac{7}{5}$ and $\frac{13}{5}$ on the number line. Write each fraction as a mixed number.



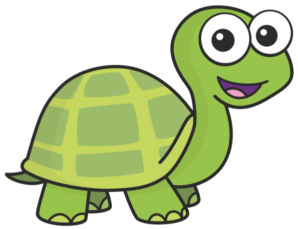
- **Dora** $\frac{14}{4} = 3\frac{2}{4}$ **Mo** $\frac{14}{4} = 3\frac{1}{2}$

Whose conversion of $\frac{14}{4}$ do you agree with?

Convert improper fractions to mixed numbers

Reasoning and problem solving

$\frac{28}{3}$ is less than $\frac{37}{5}$ because 28 is less than 37



Do you agree with Tiny?

Explain your answer.

No

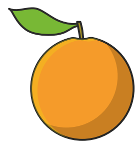
Which is greater, $\frac{19}{3}$ or $\frac{25}{4}$?

Explain your answer.

$\frac{19}{3}$

At half-time in a netball match, all seven players and both reserves are given half an orange.

How many oranges are needed altogether?



5 (there will be half an orange left over)

Work out the missing numbers.

$\frac{23}{4} = \square \frac{\square}{\square}$

23 ÷ 4 = ______ remainder ______

What do you notice?

$5\frac{3}{4}$

5 remainder 3

Convert mixed numbers to improper fractions

Notes and guidance

This small step builds on the previous step and relies on children's understanding of the whole.

Children convert from mixed numbers to improper fractions by identifying how many of the equal parts each whole is worth and using this to work out how many equal parts are needed for the integer part of the mixed number. They then add on the number of parts in the fractional part of the mixed number and finally write the answer as an improper fraction.

As in the previous step, cubes, bar models and other representations can be used to support children's understanding.

Things to look out for

- Children may not make connections between the denominator of a fraction and the number of equal parts needed to make one whole.
- Instead of using multiplication, children may count each of the individual parts, which is inefficient and likely to lead to errors.
- Children may have the misconception that a fraction is always "part of a whole" and not realise that an improper fraction is a fraction that is greater than 1

Key questions

- How many ______ are there in one whole?
- How many ______ are there in ______ wholes?
- How many ______ are there altogether in the mixed number? How can you write this as an improper fraction?
- How many cubes do you need to represent the mixed number? How many cubes do you need for each whole? How many more cubes do you need? How many cubes do you need altogether?

Possible sentence stems

- Each whole can be split into ______
- The wholes are equal to ______ altogether.
- There are another ______ so the mixed number is ______ as an improper fraction.

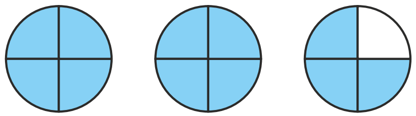
National Curriculum links

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Convert mixed numbers to improper fractions

Key learning

- Each circle represents one whole.

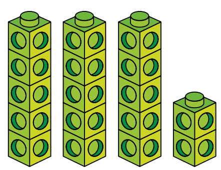


  - What mixed number does the diagram show?
  - What improper fraction does the diagram show?

- Each bar model represents one whole.

  - What mixed number does the bar model show?
  - What improper fraction does the bar model show?

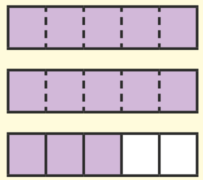
- Kim uses cubes to help convert $3\frac{2}{5}$ to an improper fraction.



  - Why has Kim built 3 towers of 5 cubes?
  - Why are there only 2 cubes in the fourth tower?
  - How many cubes has Kim used altogether?
  - Write $3\frac{2}{5}$ as an improper fraction.

- Tom uses a bar model to convert $2\frac{3}{5}$ to an improper fraction.

  Complete Tom's workings.



$2\frac{3}{5}$ = ______ wholes + ______ fifths

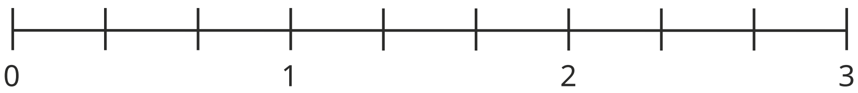
2 wholes = ______ fifths

______ fifths + ______ fifths = ______ fifths

$2\frac{3}{5} = \frac{\square}{5}$

  Use Tom's method to convert $3\frac{2}{3}$, $2\frac{5}{6}$, $4\frac{3}{4}$ and $7\frac{1}{2}$ to improper fractions.

- Show $1\frac{2}{3}$ and $2\frac{1}{3}$ on the number line.



  Write each mixed number as an improper fraction.

  Use number lines to convert $3\frac{3}{4}$ and $3\frac{3}{5}$ to improper fractions.

- Which is greater, $4\frac{1}{4}$ or $\frac{21}{5}$?

Convert mixed numbers to improper fractions

Reasoning and problem solving

All the children in a class eat $\frac{1}{3}$ of a pizza at a party.

Altogether, they eat $8\frac{2}{3}$ pizzas.

How many children are there in the class?

26 children

Each episode of Dexter's favourite cartoon is $\frac{1}{4}$ hour long.

One day, Dexter watches his favourite cartoon for $2\frac{3}{4}$ hours without stopping!

How many episodes does Dexter watch?

How many episodes could Dexter watch in $4\frac{1}{2}$ hours?

11 episodes

18 episodes

How many different ways can you complete the statements?

$$2\frac{\square}{8} = \frac{\square}{8}$$

$$2\frac{\square}{5} = \frac{\square}{5}$$

Compare answers with a partner.

What do you notice?

seven possibilities

$2\frac{1}{8} = \frac{17}{8}$ $2\frac{2}{8} = \frac{18}{8}$

$2\frac{3}{8} = \frac{19}{8}$ $2\frac{4}{8} = \frac{20}{8}$

$2\frac{5}{8} = \frac{21}{8}$ $2\frac{6}{8} = \frac{22}{8}$

$2\frac{7}{8} = \frac{23}{8}$

four possibilities

$2\frac{1}{5} = \frac{11}{5}$ $2\frac{2}{5} = \frac{12}{5}$

$2\frac{3}{5} = \frac{13}{5}$ $2\frac{4}{5} = \frac{14}{5}$

Compare fractions less than 1

Notes and guidance

Building on their knowledge of equivalent fractions, in this small step children compare fractions where the denominators are the same or where one denominator is a multiple of the other. They also compare fractions with the same numerator or by considering their position relative to one half.

Diagrams will help children to see which is the larger fraction and they should continue to use fraction walls and bar models until they are confident with the general rules.

The next step builds on this knowledge, with children ordering sets of fractions using the same techniques. They will also use equal denominators when adding and subtracting fractions and mixed numbers later in the block.

Things to look out for

- Children may mix up the meanings of the > and < symbols. In particular, they sometimes do not realise that statements such as $\frac{1}{2} > \frac{1}{3}$ and $\frac{1}{3} < \frac{1}{2}$ say the same thing even though the signs are different.
- When comparing fractions, children may focus solely on the numerators or the denominators.
- Children may incorrectly think that since 7 > 6, $\frac{5}{7} > \frac{5}{6}$

Key questions

- If two fractions have the same denominator/numerator, how can you decide which is greater?
- How can you use equivalent fractions to help?
- How can you use a diagram to find equivalent fractions? Do the bars need to be the same size?

Possible sentence stems

- ______ is greater than one half, and ______ is less than one half, so ______ is greater than ______
- When two fractions have the same denominator, the one with the ______ numerator is the greater fraction.
- When two fractions have the same numerator, the one with the ______ denominator is the greater fraction.

National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

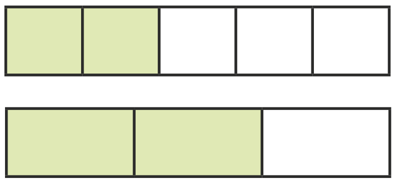
Compare fractions less than 1

Key learning

- Use diagrams to show that $\frac{4}{5} > \frac{3}{5}$

  Explain how you can tell that $\frac{4}{5} > \frac{3}{5}$ without using a diagram.

- Use the bar models to compare the fractions.

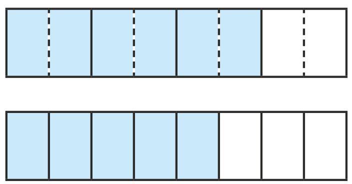


$\frac{2}{5}$ ◯ $\frac{2}{3}$

$\frac{2}{3}$ ◯ $\frac{2}{5}$

- Identify the greater fraction in each pair.
  - $\frac{3}{7}$ and $\frac{5}{7}$
  - $\frac{9}{11}$ and $\frac{2}{11}$
  - $\frac{2}{3}$ and $\frac{2}{15}$
  - $\frac{1}{8}$ and $\frac{1}{3}$
  - $\frac{4}{15}$ and $\frac{4}{100}$
  - $\frac{11}{20}$ and $\frac{19}{20}$

- Tommy uses bar models and equivalent fractions to compare $\frac{3}{4}$ and $\frac{5}{8}$



  Which is the greater fraction, $\frac{3}{4}$ or $\frac{5}{8}$?

  How do you know?

- Decide which is the greater fraction in each pair.

$\frac{5}{6}$ and $\frac{2}{3}$	$\frac{2}{3}$ and $\frac{5}{9}$	$\frac{7}{16}$ and $\frac{3}{8}$

- Esther and Aisha each have a chocolate bar.

  Esther has eaten $\frac{3}{5}$ of her chocolate bar.

  Aisha has eaten $\frac{8}{15}$ of her chocolate bar.

  Who has eaten more of their chocolate bar?

- Write <, > or = to compare the fractions.

$\frac{7}{8}$ ◯ $\frac{3}{4}$　　$\frac{6}{14}$ ◯ $\frac{3}{7}$

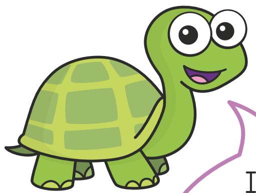
$\frac{9}{10}$ ◯ $\frac{9}{100}$　　$\frac{1}{4}$ ◯ $\frac{1}{40}$

$\frac{5}{9}$ ◯ $\frac{9}{9}$　　$\frac{1}{5}$ ◯ $\frac{5}{20}$

Compare fractions less than 1

Reasoning and problem solving

Tiny is comparing $\frac{7}{10}$ and $\frac{3}{8}$



How can Tiny use these facts to compare $\frac{7}{10}$ and $\frac{3}{8}$?

Use Tiny's method to compare the fractions.

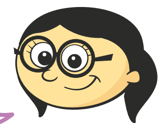
$\frac{17}{28}$ ◯ $\frac{11}{32}$

$\frac{999}{2000}$ ◯ $\frac{3007}{5000}$

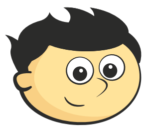
$>$

$<$

Annie and Jack are using a number line to work out which fraction is greater, $\frac{5}{6}$ or $\frac{2}{3}$



Annie



Jack

Will Annie and Jack get the same answer?

Show working to support your answer.

Which method do you prefer?

Yes

Order fractions less than 1

Notes and guidance

In this small step, children build on their knowledge from the previous step to order a set of three or more fractions.

If equivalent fractions are needed, then one denominator will be a multiple of the other(s) so that conversions will not be complicated. Children will not need to compare fractions such as $\frac{2}{5}$ and $\frac{3}{7}$ until Year 6

Bar models, fraction walls and number lines will still be useful to help children to see the relative sizes of the fractions, especially when conversions are needed. Children should look at the set of fractions as a whole before deciding their approach, as comparing numerators could still be a better strategy for some sets of fractions.

Children can use other strategies covered in the previous step, such as considering the position of a fraction relative to 0, $\frac{1}{2}$ or 1 whole.

Things to look out for

- At first, children may need support to decide the best strategy when there are more than two fractions.
- Children may not look at both parts of the fractions when making their decisions about the order.

Key questions

- If a set of fractions all have the same denominator, how can you tell which is greatest?
- If a set of fractions all have the same numerator, how can you tell which is greatest?
- How can you use equivalent fractions to help?
- What are all the denominators/numerators multiples of? How can this help you find equivalent fractions?
- Which of the fractions are greater than $\frac{1}{2}$?

Possible sentence stems

- When fractions have the same denominator, the one with the _______ numerator is the greatest fraction.
- When fractions have the same numerator, the one with the _______ denominator is the greatest fraction.

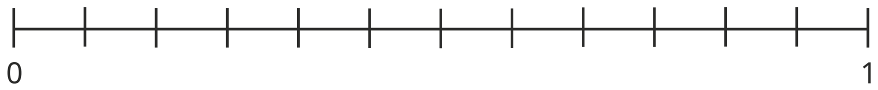
National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Order fractions less than 1

Key learning

- Label $\frac{1}{12}$, $\frac{11}{12}$ and $\frac{7}{12}$ on the number line.



Write the three fractions in order of size, starting with the smallest.

- Complete the equivalent fractions.

$\frac{1}{2} = \frac{\square}{12}$ $\frac{3}{4} = \frac{\square}{12}$

Use your answers to help you write these fractions in order of size, starting with the smallest.

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{1}{12}$

- Use equivalent fractions to write the fractions in ascending order.

$\frac{3}{5}$	$\frac{1}{2}$	$\frac{7}{10}$

- Order each set of fractions, from greatest to smallest.
  - $\frac{3}{7}, \frac{3}{5}, \frac{3}{8}$
  - $\frac{2}{3}, \frac{5}{6}, \frac{7}{12}$
  - $\frac{1}{4}, \frac{2}{5}, \frac{3}{20}$

- Brett, Dani and Huan take the same spelling test.

Brett gets $\frac{3}{4}$ of the spellings correct.

Dani gets $\frac{3}{5}$ of the spellings correct.

Huan gets 19 of the 40 spellings correct.

  - Who got the most spellings correct?
  - Who got the least spellings correct?

- Order each set of fractions, from smallest to greatest.

$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{9}$

$\frac{7}{9}$	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{6}{9}$	$\frac{9}{9}$

$\frac{9}{20}$	$\frac{9}{10}$	$\frac{9}{100}$	$\frac{9}{1000}$	$\frac{9}{15}$	$\frac{9}{40}$

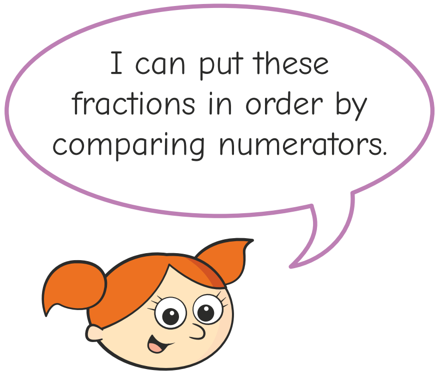
- Order the fractions using the > symbol.

$\frac{5}{11}$	$\frac{5}{12}$	$\frac{6}{11}$

Order fractions less than 1

Reasoning and problem solving

Alex is ordering fractions.



$\frac{8}{15}$ $\frac{2}{7}$ $\frac{4}{11}$

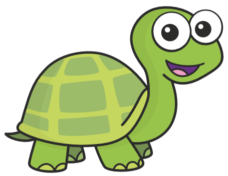
Explain how Alex can do this.

List the fractions in order, starting with the smallest.

8 and 4 are both multiples of 2, so Alex can rewrite the fractions such that all have a numerator of 8

As $\frac{8}{28} < \frac{8}{22} < \frac{8}{15}$, the correct order is $\frac{2}{7}, \frac{4}{11}, \frac{8}{15}$

Tiny is ordering some fractions.



$\frac{1}{2} < \frac{2}{5} < \frac{3}{10} < \frac{7}{8}$

Explain the mistake Tiny has made.

Tiny has only looked at the numerators.

Fill in the boxes to make the statement true.

$\frac{3}{8} < \frac{\square}{\square} < \frac{3}{4}$

Complete the statement in two different ways.

Compare answers with a partner.

multiple possible answers, e.g. $\frac{4}{8}, \frac{5}{8}, \frac{3}{7}, \frac{3}{6}, \frac{3}{5}$

Compare and order fractions greater than 1

Notes and guidance

In this small step, children consolidate their knowledge from all the earlier steps in this block, using their skills in converting between forms to help compare and order fractions greater than 1

Children understand that if the number of wholes is different, they do not need to compare the fractional parts. When the number of wholes is equal, they compare denominators or numerators of the fractional parts. As with earlier steps, such comparisons will be limited to instances where the numerators or denominators are equal, or one denominator or numerator is a multiple of the other.

Again, diagrams will be helpful for students to see the comparative sizes of the numbers.

Things to look out for

- Children may need support to see the most efficient strategy, for example deciding whether any conversion is needed and/or whether to compare denominators or numerators.
- Errors may occur when converting between mixed numbers and improper fractions.

Key questions

- How can you represent the fractions?
- What does the number of wholes tell you about the overall sizes of the numbers?
- Do you need to make any conversions?
- How do you convert from an improper fraction/mixed number to a mixed number/improper fraction?
- How can you use your knowledge of multiples to help?

Possible sentence stems

- If the number of wholes is different, then the number with _______ wholes is greater.
- If the number of wholes is equal, then I need to compare the _______

National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Compare and order fractions greater than 1

Key learning

- Draw a diagram to show how you know that $3\frac{1}{3} > 2\frac{1}{2}$

- Write < or > to compare the numbers.

  $4\frac{1}{2}$ ◯ $3\frac{1}{2}$ $5\frac{1}{3}$ ◯ 4

  $2\frac{4}{5}$ ◯ $3\frac{1}{4}$ 3 ◯ $4\frac{1}{3}$

- Put the numbers in order, starting with the smallest.

  $3\frac{1}{3}$ $5\frac{1}{2}$ $2\frac{3}{4}$ $4\frac{1}{5}$ $1\frac{9}{10}$

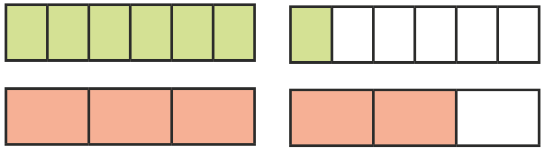
- Explain how the diagrams show that $2\frac{1}{3} < 2\frac{1}{2}$

- Write < or > to compare the numbers.

  $4\frac{1}{2}$ ◯ $4\frac{1}{5}$ $2\frac{4}{5}$ ◯ $2\frac{4}{7}$

  $5\frac{3}{4}$ ◯ $5\frac{1}{4}$ $3\frac{5}{8}$ ◯ $3\frac{7}{8}$

- Use the bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$



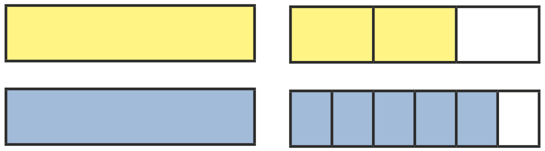
  ______ > ______

  ______ < ______

  In each pair of fractions, decide which fraction is greater.

  ▸ $\frac{5}{2}$ and $\frac{9}{4}$ ▸ $\frac{11}{6}$ and $\frac{5}{3}$ ▸ $\frac{9}{4}$ and $\frac{17}{8}$

- Use the bar models to compare $1\frac{2}{3}$ and $1\frac{5}{6}$



  ______ < ______

  ______ > ______

  In each pair of mixed numbers, decide which is greater.

  ▸ $1\frac{5}{8}$ and $1\frac{1}{2}$ ▸ $2\frac{3}{7}$ and $2\frac{9}{14}$ ▸ $3\frac{3}{4}$ and $3\frac{7}{8}$

- Use common denominators to put each set of numbers in order, starting with the smallest.

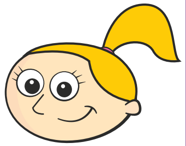
$\frac{8}{5}$	$\frac{11}{10}$	$\frac{17}{20}$

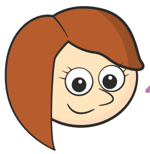
$1\frac{2}{3}$	$1\frac{7}{24}$	$\frac{11}{12}$

Compare and order fractions greater than 1

Reasoning and problem solving

Eva and Rosie each have two identical pizzas.

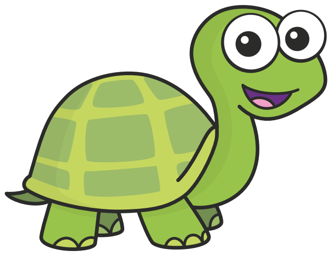




Who has eaten more pizza?

Explain how you know.

Rosie



$$2\frac{3}{4} < \square < \frac{10}{3}$$

What mixed numbers and improper fractions can Tiny find?

Compare answers with a partner.

multiple possible answers, e.g.

$2\frac{7}{8} = \frac{21}{8}$

$2\frac{9}{10} = \frac{29}{10}$

$3\frac{1}{10} = \frac{31}{10}$

$3\frac{1}{5} = \frac{16}{5}$

$3\frac{1}{4} = \frac{13}{4}$

Add and subtract fractions with the same denominator

Notes and guidance

In this small step, children add and subtract fractions with the same denominator. For adding, this will include adding three or more fractions as well as pairs of fractions.

Children need to understand that when the denominators are the same, they only need to add or subtract the numerators. Bar models are a useful way of representing both addition and subtraction of fractions and are easier to work with than circles, as they are easier to draw and easier to split into equal parts.

Now that children are comfortable working with improper fractions, these are incorporated into both the questions and answers of this small step. For some questions, answers could be written in a simplified form, but this is not the focus of the step.

Things to look out for

- A common error is to add or subtract both the numerators and the denominators, for example $\frac{3}{4} + \frac{3}{4} = \frac{6}{8}$ or $\frac{5}{6} - \frac{1}{6} = \frac{4}{0}$
- Children may not look at the symbols in the calculations and, for example, add the fractions when the intended calculation is a subtraction.

Key questions

- How can you represent this calculation using a bar model?
- Will you need more than one bar? How do you know?
- How many parts do you split the bar(s) into?
- What could you do if the answer is an improper fraction?
- What type of calculation is this?
- When adding/subtracting fractions with the same denominators, what will the denominator of the answer be? Why?

Possible sentence stems

- ______ fifths add/subtract ______ fifths is ______ fifths.
- When adding/subtracting fractions with the same denominators, I just add/subtract the ______

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

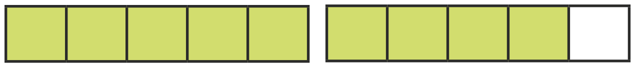
Add and subtract fractions with the same denominator

Key learning

- Use the bar model to complete the calculation.

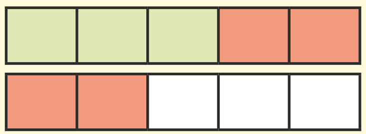
$\frac{1}{5} + \frac{2}{5} =$ ______

- Use the bar model to help you work out $\frac{9}{5} - \frac{6}{5}$



- Work out the calculations.
  - $\frac{4}{9} + \frac{1}{9}$
  - $\frac{3}{7} + \frac{2}{7}$
  - $\frac{4}{9} + \frac{1}{9} + \frac{2}{9}$
  - $\frac{7}{9} - \frac{5}{9}$
  - $\frac{1}{8} + \frac{3}{8} + \frac{3}{8}$
  - $\frac{6}{7} - \frac{4}{7}$

- Teddy uses a bar model to work out $\frac{3}{5} + \frac{4}{5}$



$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$

Use bar models to find the totals.

$\frac{2}{3} + \frac{2}{3}$ | $\frac{3}{4} + \frac{3}{4} + \frac{1}{4}$ | $\frac{6}{5} + \frac{4}{5}$

- Work out the missing numbers.
  - $\frac{3}{7} + \frac{\square}{7} = \frac{9}{7}$
  - $\frac{3}{11} + \frac{\square}{11} = \frac{15}{11}$
  - $\frac{15}{8} - \frac{\square}{8} = 1$
  - $\frac{\square}{5} - \frac{3}{5} = \frac{4}{5}$
  - $\frac{4}{5} + \frac{\square}{5} = 2$
  - $\frac{10}{3} - \frac{\square}{3} = 2$

- A flag is made from different-coloured parts.
  - $\frac{2}{15}$ of the flag is blue.
  - $\frac{7}{15}$ of the flag is red.
  - $\frac{1}{15}$ of the flag is green.

  - What fraction of the flag is blue, red or green?

  The rest of the flag is white.

  - What fraction of the flag is white?

- Work out the missing numbers.
  - $\frac{3}{7} + \frac{5}{7} = \frac{\square}{\square} + \frac{4}{7}$
  - $\frac{9}{5} - \frac{5}{5} = \frac{6}{5} - \frac{\square}{\square}$
  - $\frac{2}{3} + \frac{\square}{\square} = \frac{11}{3} - \frac{4}{3}$

Add and subtract fractions with the same denominator

Reasoning and problem solving

Find as many ways as you can to make the statement correct.

$$\frac{5}{9} + \frac{\square}{9} = \frac{8}{9} + \frac{\square}{9}$$

Compare answers with a partner.

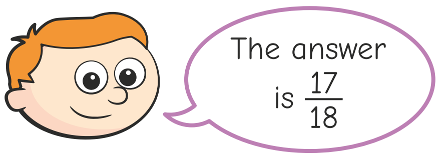
multiple possible answers, e.g.

$\frac{5}{9} + \frac{4}{9} = \frac{8}{9} + \frac{1}{9}$

$\frac{5}{9} + \frac{5}{9} = \frac{8}{9} + \frac{2}{9}$

$\frac{5}{9} + \frac{6}{9} = \frac{8}{9} + \frac{3}{9}$

Ron adds three fractions with the same denominator.



What three fractions could he have added?

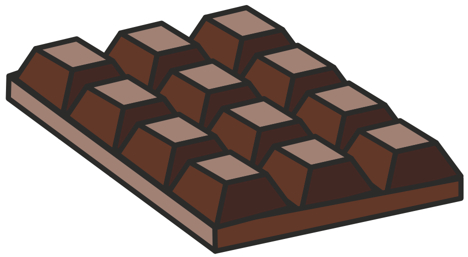
multiple possible answers, e.g.

$\frac{1}{18} + \frac{3}{18} + \frac{13}{18}$

$\frac{1}{18} + \frac{5}{18} + \frac{11}{18}$

$\frac{6}{18} + \frac{3}{18} + \frac{8}{18}$

A chocolate bar has 12 equal pieces.



- Amir eats $\frac{5}{12}$ more of the bar than Whitney.
- There is $\frac{1}{12}$ of the bar left.

What fraction of the bar does Amir eat?

What fraction of the bar does Whitney eat?

$\frac{8}{12}$

$\frac{3}{12}$

Add fractions within 1

Notes and guidance

In this small step, children add two or three fractions with different denominators. The fractions are such that one denominator is a multiple of another and the total remains within 1. Children may be familiar with some simple common additions, such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and this could be a good example on which to build.

Children can use pictorial representations to convert one of the fractions so they have a common denominator and to support the addition. If they write their workings alongside the pictorial representations, this will help them to make the links.

Things to look out for

- Children may not realise the need to make the denominators equal before adding.
- Children may add both the numerators and the denominators, for example $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$
- Children may make mistakes when finding equivalent fractions.
- Children may think that the only way to find a common denominator is to find the product of the denominators in the question.

Key questions

- Do the fractions have the same denominator?
- What does it mean for two fractions to be equivalent?
- How can you tell when two fractions are equivalent?
- Why do the denominators need to be the same?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add the fractions?

Possible sentence stems

- Fractions must have the same ______ before you can add them.
- The denominator has been multiplied by ______, so the numerator needs to be multiplied by ______ for the fractions to be equivalent.

National Curriculum links

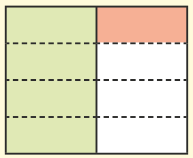
- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number

Add fractions within 1

Key learning

- Scott is working out $\frac{1}{2} + \frac{1}{8}$

  He uses a diagram to represent the sum.



$$\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

  Use Scott's method to work out the additions.

  - $\frac{1}{2} + \frac{3}{8}$
  - $\frac{1}{4} + \frac{5}{8}$
  - $\frac{7}{10} + \frac{1}{5}$

- Draw a diagram to show that $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

- Rosie uses a bar model to work out $\frac{1}{4} + \frac{3}{8}$

$$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

  Use a bar model to work out the additions.

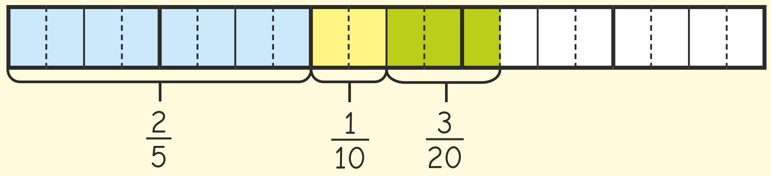
  - $\frac{1}{6} + \frac{5}{12}$
  - $\frac{2}{9} + \frac{1}{3}$
  - $\frac{1}{3} + \frac{4}{15}$

- What common denominator would you use to work out each addition?

  $\frac{1}{2} + \frac{1}{6}$ | $\frac{1}{4} + \frac{5}{8}$ | $\frac{3}{10} + \frac{2}{5}$ | $\frac{1}{6} + \frac{1}{3}$

  Work out the additions.

- Nijah uses a bar model to work out $\frac{2}{5} + \frac{1}{10} + \frac{3}{20}$



$$\frac{2}{5} + \frac{1}{10} + \frac{3}{20} = \frac{8}{20} + \frac{2}{20} + \frac{3}{20} = \frac{13}{20}$$

  Work out the additions.

  $\frac{1}{4} + \frac{3}{8} + \frac{5}{16}$ | $\frac{1}{2} + \frac{1}{6} + \frac{1}{12}$ | $\frac{1}{3} + \frac{2}{9} + \frac{5}{18}$

- Work out the missing numbers.

  - $\frac{1}{5} + \frac{\square}{10} + \frac{8}{20} = 1$
  - $\frac{1}{5} + \frac{\square}{15} + \frac{1}{30} = 1$

Add fractions within 1

Reasoning and problem solving

Work out the missing numbers.

$$\frac{5}{16} + \frac{\square}{8} = \frac{15}{16}$$

$$\frac{\square}{20} + \frac{7}{10} = \frac{17}{20}$$

5, 3

Tommy adds three fractions with different denominators.

What three fractions could he have added?

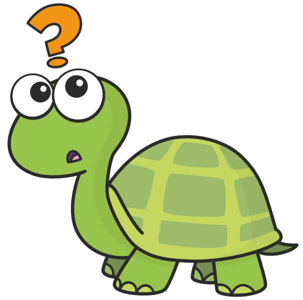
multiple possible answers, e.g.

$\frac{1}{18} + \frac{2}{9} + \frac{2}{3}$

$\frac{3}{18} + \frac{4}{9} + \frac{1}{3}$

Tiny is trying to work out this addition.

$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20}$$



$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20} = \frac{7}{35}$$

What mistake has Tiny made?

Find the correct total.

$\frac{17}{20}$

Add fractions with total greater than 1

Notes and guidance

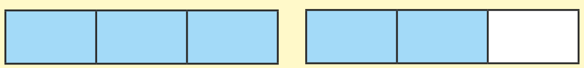
In this small step, children continue to add fractions where one denominator is a multiple of the other, but progress to additions where the total is greater than 1. Their answers will be improper fractions that they should convert to mixed numbers using the skills they have learnt in earlier steps.

Children continue to represent the addition of fractions using pictorial or concrete representations to make sense of both the methods and the answers.

They need to be clear what represents the whole in each case.

Things to look out for

- Children may forget to find a common denominator before adding the fractions.
- Children may add the denominators as well as the numerators.
- Children may misinterpret their diagrams, not realising that the answer is more than one whole, for example interpreting this diagram as $\frac{5}{6}$ instead of $1\frac{2}{3}$



Key questions

- Do the fractions have the same denominator?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add the fractions?
- How can you tell the answer is greater than one whole?
- How can you convert the answer to a mixed number?

Possible sentence stems

- I am going to make all of the denominators ______
- I can regroup ______ to make ______ wholes with ______ parts left over. As a mixed number, this is ______ and ______

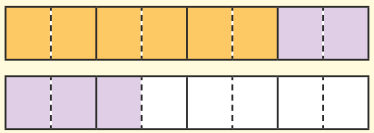
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add fractions with total greater than 1

Key learning

- Dora uses a bar model to add $\frac{3}{4}$ and $\frac{5}{8}$



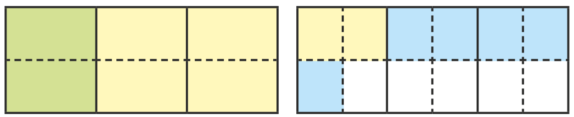
$\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}$

Use Dora's method to work out the additions.

Give your answers as mixed numbers.

▸ $\frac{2}{3} + \frac{5}{12}$ ▸ $\frac{5}{8} + \frac{11}{16}$ ▸ $\frac{7}{10} + \frac{3}{5}$

- Here is Mo's method for adding three fractions with different denominators.



$\frac{1}{3} + \frac{5}{6} + \frac{5}{12} = 1\frac{7}{12}$

Explain each step of the calculation.

Add the sets of fractions, giving your answers as mixed numbers.

▸ $\frac{2}{3} + \frac{1}{6} + \frac{7}{12}$ ▸ $\frac{1}{4} + \frac{7}{8} + \frac{3}{16}$ ▸ $\frac{1}{2} + \frac{5}{6} + \frac{5}{12}$

- Use the bar model to work out $\frac{3}{4} + \frac{3}{8} + \frac{1}{2}$

Give your answer as a mixed number.

Draw your own bar models to add the fractions, giving your answers as mixed numbers.

$\frac{5}{12} + \frac{1}{6} + \frac{1}{2}$ $\frac{11}{20} + \frac{3}{5} + \frac{1}{10}$ $\frac{3}{4} + \frac{5}{12} + \frac{1}{2}$

- What common denominator would you use to work out these additions?

$\frac{1}{2} + \frac{5}{6} + \frac{7}{12}$ $\frac{2}{5} + \frac{7}{20} + \frac{9}{10}$ $\frac{4}{15} + \frac{2}{3} + \frac{11}{30}$

Work out the additions.

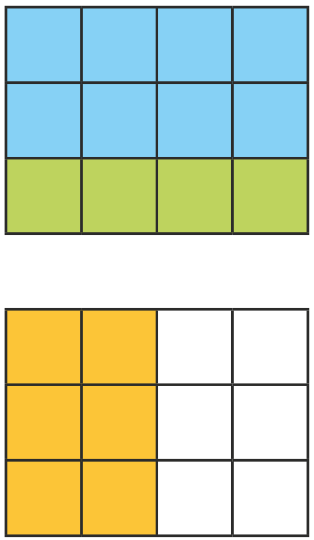
- Work out the missing numbers.

▸ $\frac{7}{10} + \frac{\square}{5} = 1\frac{3}{10}$ ▸ $\frac{3}{4} + \frac{7}{8} + \frac{\square}{8} = 2$ ▸ $3\frac{1}{12} = \frac{\square}{12} + \frac{2}{3} + \frac{5}{6}$

Add fractions with total greater than 1

Reasoning and problem solving

Kim uses the diagram to add three fractions.



What could her fractions be?

How many different combinations can you find?

multiple possible answers, e.g.

$\frac{2}{3} + \frac{4}{12} + \frac{1}{2}$

$\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square} = 2\frac{1}{8}$

The sum of three fractions is $2\frac{1}{8}$

Use the clues to work out the three fractions.

- All the fractions have different denominators.
- None of the fractions are smaller than one half.
- None of the fractions are improper.
- All three denominators are factors of 8

Investigate different sets of fractions that have the same total but satisfy fewer of the clues.

$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$

Add to a mixed number

Notes and guidance

In this small step, children add either a whole number part or a fractional part to a mixed number as a precursor to adding two mixed numbers in the next step.

The key point is that children remember that a mixed number such as $3\frac{1}{2}$ can be partitioned into $3 + \frac{1}{2}$ and then they can add to the required part before recombining. The expectation is that, provided the sum of fractional parts does not cross a whole, these additions will generally be done mentally. Pictorial support may still be useful, initially.

Crossing the whole will be included towards the end of this step and will feature more prominently in the next step.

Things to look out for

- Children may need to be reminded how to partition mixed numbers.
- Children may omit one or more of the numbers being added in a calculation and not recombine all the parts into the correct mixed number at the end.
- When totals cross a whole, children need to be confident converting improper fractions. Recombining will require careful recording to prevent slips.

Key questions

- How can you partition a mixed number?
- How can the addition be written so that similar parts are next to each other?
- How can the parts be combined to produce a mixed number?
- Do you need to combine whole numbers or fractions?
- Why can you swap the order of the numbers in an addition?

Possible sentence stems

- A mixed number can be partitioned into a ______ part and a ______ part.
- The fractional part of the answer is an ______, so needs converting to a ______

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add to a mixed number

Key learning

- Tom adds a fraction to a mixed number by adding the fractions separately and then adding the wholes.

  $4\frac{2}{9} + \frac{3}{9}$

  $4 + \frac{2}{9} + \frac{3}{9}$

  $4 + \frac{5}{9}$

  $4\frac{5}{9}$

  Use Tom's method to work out the additions.

  $3\frac{1}{5} + \frac{3}{5}$ | $4\frac{1}{3} + \frac{1}{3}$ | $\frac{2}{7} + 3\frac{4}{7}$ | $\frac{2}{9} + 3\frac{5}{9}$

- Here is Esther's method for working out $2\frac{3}{4} + 3$

  $2\frac{3}{4} + 3$

  $2 + \frac{3}{4} + 3$

  $2 + 3 + \frac{3}{4}$

  $5 + \frac{3}{4}$

  $5\frac{3}{4}$

  Use Esther's method to work out the additions.

  $3\frac{1}{2} + 2$ | $6 + 2\frac{1}{5}$ | $5\frac{3}{7} + 2$ | $3 + 2\frac{4}{5}$

- Aisha thinks $4\frac{3}{8} + \frac{1}{8} = 4\frac{4}{8}$

  Dani thinks $4\frac{3}{8} + \frac{1}{8} = 4\frac{1}{2}$

  Who do you agree with?

- Work out the totals.

  $3\frac{2}{9} + \frac{4}{9}$ | $2\frac{5}{8} + \frac{1}{8}$ | $\frac{5}{12} + 4\frac{5}{12}$

- Kim uses equivalent fractions to work out $4\frac{1}{3} + \frac{1}{6}$

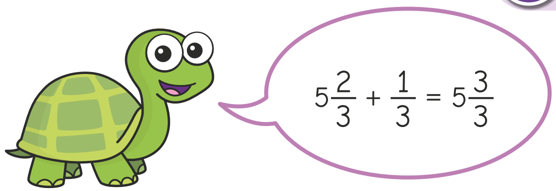
  $$4\frac{1}{3} + \frac{1}{6} = 4 + \frac{1}{3} + \frac{1}{6}$$
  $$= 4 + \frac{2}{6} + \frac{1}{6}$$
  $$= 4\frac{3}{6}$$
  $$= 4\frac{1}{2}$$

  Work out the totals.

  $3\frac{2}{5} + \frac{3}{10}$ | $2\frac{7}{12} + \frac{1}{6}$ | $\frac{3}{8} + 3\frac{3}{16}$ | $1\frac{2}{5} + \frac{4}{15}$

Add to a mixed number

Reasoning and problem solving



How can you improve on Tiny's answer?

Here are some additions.

$\frac{5}{12} + 2\frac{7}{12}$ | $6\frac{3}{5} + \frac{2}{5}$ | $6\frac{3}{5} + \frac{3}{5}$

$\frac{4}{7} + 2\frac{5}{7}$ | $3\frac{7}{8} + \frac{5}{8}$

Work out the totals.

Think carefully about how you give your answers.

$3,\ 7,\ 7\frac{1}{5}$

$3\frac{2}{7},\ 4\frac{1}{2}$

Here are some fraction cards.

$\frac{4}{5}$ | $\frac{2}{3}$ | $\frac{1}{4}$

Decide which fraction card belongs in which calculation.

$4\frac{7}{9} = 4\frac{1}{9} + \square$

$3\frac{3}{8} + \square = 3\frac{5}{8}$

$\frac{43}{10} = 3\frac{1}{2} + \square$

$\frac{2}{3}$

$\frac{1}{4}$

$\frac{4}{5}$

What could the values of

A and B be?

$A\frac{5}{12} + \frac{B}{4} = 5\frac{1}{6}$

Compare answers with a partner.

multiple possible answers, e.g.

A = 4 and B = 3

Add two mixed numbers

Notes and guidance

Building on the previous step, children add two mixed numbers by adding the wholes and fractional parts separately. This is usually the most efficient method of adding two mixed numbers, but converting to improper fractions and adding them is included as an alternative. Examples are included where children need to use equivalent fractions and where answers can be simplified, although simplifying answers is not the priority here.

Children can still draw models to represent adding fractions, particularly if these are useful for pairs of fractions with different denominators. The cognitive load is significant when finding solutions to these multi-step problems, so providing scaffolding/ partially started solutions may be useful.

Things to look out for

- Children may make errors in the partitioning or recombining of the integer and fractional parts.
- Arithmetical errors may occur when converting to improper fractions with larger numbers.
- Where the fractional parts cross the whole, children may not interpret this correctly, either leaving their answer as a whole and improper fraction, or converting but not adding 1 to the integer.

Key questions

- How can you partition the mixed numbers?
- How can the addition be rewritten to make it easier?
- Do you need to combine whole numbers, fractions or both?
- Are there any improper fractions in the answer? What can you do about this?
- How do you change a mixed number into an improper fraction?
- In this question, is it easier to deal with mixed numbers or to use improper fractions? Why?

Possible sentence stems

- The mixed numbers can be partitioned into a ______ part and a ______ part.

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add two mixed numbers

Key learning

- Use bar models to show that $2\frac{2}{5} + 3\frac{1}{5} = 5\frac{3}{5}$
- Annie adds two mixed numbers by adding the wholes first and then adding the fractions.

$$2\frac{3}{5} + 4\frac{1}{5} = 6 + \frac{4}{5} = 6\frac{4}{5}$$

Use Annie's method to find the totals.

▸ $3\frac{4}{9} + 2\frac{1}{9}$ ▸ $1\frac{2}{7} + 4\frac{3}{7}$ ▸ $5\frac{11}{15} + 3\frac{2}{15}$

- Use bar models to show that $2\frac{1}{2} + 3\frac{1}{4} = 5\frac{3}{4}$
- Filip uses equivalent fractions to add $1\frac{1}{3}$ and $2\frac{1}{6}$

$$1\frac{1}{3} + 2\frac{1}{6} = 1 + 2 + \frac{1}{3} + \frac{1}{6} = 3 + \frac{2}{6} + \frac{1}{6} = 3\frac{3}{6} = 3\frac{1}{2}$$

Use Filip's method to find the totals.

▸ $3\frac{1}{4} + 2\frac{3}{8}$ ▸ $4\frac{1}{9} + 3\frac{2}{3}$ ▸ $2\frac{5}{12} + 2\frac{1}{3}$

- Jack adds $1\frac{3}{4}$ to $2\frac{1}{8}$ by converting them to improper fractions.

$$1\frac{3}{4} + 2\frac{1}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3\frac{7}{8}$$

Use Jack's method to find the totals.

▸ $1\frac{1}{4} + 2\frac{5}{12}$ ▸ $2\frac{1}{9} + 1\frac{1}{3}$ ▸ $2\frac{1}{6} + 2\frac{2}{3}$

Would you use Jack's method or a different method to find the answer to $6\frac{3}{7} + 9\frac{3}{14}$?

- Alex adds $5\frac{4}{5}$ and $4\frac{3}{5}$ by adding the wholes first and then adding the fractions.

$$5\frac{4}{5} + 4\frac{3}{5} = 9 + \frac{7}{5} = 9\frac{7}{5}$$

How can Alex's answer be improved?

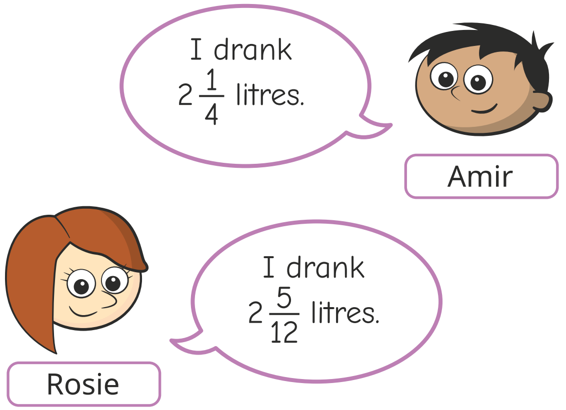
Work out the additions.

▸ $4\frac{7}{9} + 2\frac{1}{3}$ ▸ $2\frac{5}{6} + 1\frac{1}{3}$ ▸ $\frac{15}{8} + 2\frac{1}{4}$

Add two mixed numbers

Reasoning and problem solving

Amir and Rosie measure how much water they drink one day.



How much water do they drink altogether that day?

How many different ways can you find to work out the answer?

Which method do you think is most efficient?

Explain your answer.

$4\frac{2}{3}$ litres

Discuss preferred methods (e.g. wholes and fractions, improper fractions, diagrams) as a class.

Here are some sequences on number tracks.

$2\frac{1}{5}$	$2\frac{4}{5}$		4			

$1\frac{3}{7}$				$4\frac{2}{7}$	5

Fill in the missing numbers.

$3\frac{2}{5}$, $4\frac{3}{5}$, $5\frac{1}{5}$, $5\frac{4}{5}$

$2\frac{1}{7}$, $2\frac{6}{7}$, $3\frac{4}{7}$

Complete the addition.

$$3\frac{2}{5} + \square = \frac{81}{10}$$

$4\frac{7}{10}$

Subtract fractions

Notes and guidance

Children subtracted fractions with the same denominators earlier in this block. In this small step, they now move on to subtract fractions where one denominator is a multiple of the other, using the same skills they learned for adding fractions of this type.

Both proper and improper fractions are included, but this step does not include mixed numbers, conversions and crossing the whole; these will follow in subsequent steps. It is useful to consider subtraction in all its forms: partitioning, reduction and finding the difference. Pictorial representations such as bar models and number lines will help support understanding.

Things to look out for

- Children may not realise the need to make the denominators equal before subtracting.
- Children may subtract both the numerators and the denominators, for example $\frac{7}{8} - \frac{1}{3} = \frac{6}{5}$
- Errors may occur when finding equivalent fractions or misreading the question and adding instead of subtracting.
- Children may think that the only way to find a common denominator is to multiply the two denominators.

Key questions

- Do the fractions have the same denominator?
- When are two fractions equivalent?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you subtract the fractions?
- How can you represent the problem using a diagram?

Possible sentence stems

- Fractions must have the same ______ before they can be subtracted.
- The denominator has been multiplied by ______, so the numerator needs to be multiplied by ______ for the fractions to be equivalent.

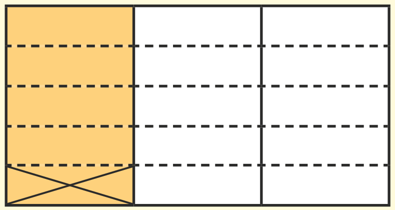
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number

Subtract fractions

Key learning

- Eva is working out $\frac{1}{3} - \frac{1}{15}$



$$\frac{1}{3} - \frac{1}{15} = \frac{5}{15} - \frac{1}{15} = \frac{4}{15}$$

Explain each step in her calculation.

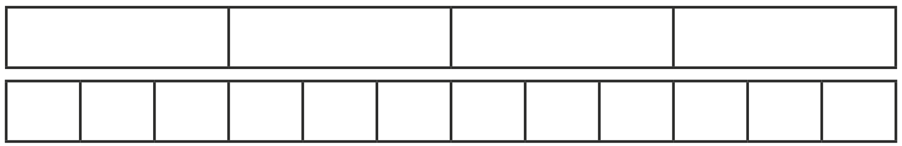
Use Eva's method to work out the subtractions.

$\frac{5}{6} - \frac{2}{3}$	$\frac{7}{8} - \frac{5}{16}$	$\frac{1}{2} - \frac{1}{10}$

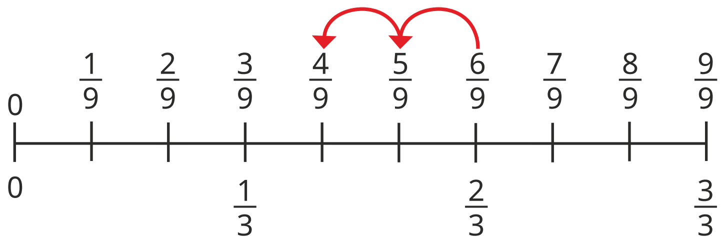
- Huan and Whitney both started with the same size chocolate bar.

Huan has $\frac{3}{4}$ of his chocolate bar left and Whitney has $\frac{5}{12}$ of her chocolate bar left.

How much more chocolate does Huan have than Whitney?



- The number line shows that $\frac{2}{3} - \frac{2}{9} = \frac{4}{9}$



Work out the subtractions.

$\frac{2}{3} - \frac{5}{9}$	$\frac{7}{9} - \frac{4}{9}$	$\frac{8}{9} - \frac{1}{3}$

- Find the difference between each pair of fractions.

$\frac{5}{12}$ and $\frac{3}{4}$	$\frac{3}{5}$ and $\frac{19}{15}$	$\frac{20}{9}$ and $\frac{4}{3}$

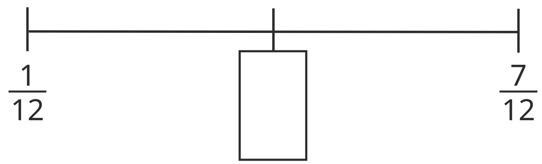
- Work out the subtractions.

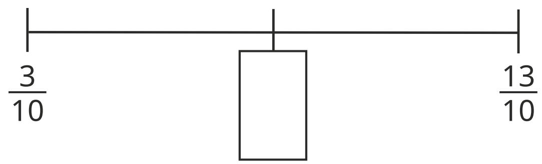
  - $\frac{3}{4} - \frac{5}{16}$
  - $\frac{7}{3} - \frac{8}{9}$
  - $\frac{11}{12} - \frac{2}{3}$
  - $\frac{17}{20} - \frac{4}{5}$
  - $\frac{12}{5} - \frac{7}{10}$
  - $\frac{14}{14} - \frac{11}{14}$

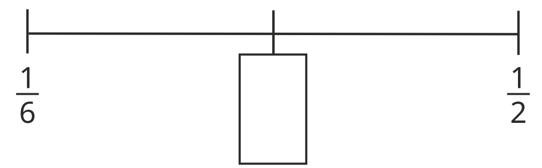
Subtract fractions

Reasoning and problem solving

Find the value of the midpoint of each number line.







$\frac{4}{12}$ (or $\frac{1}{3}$)

$\frac{8}{10}$ (or $\frac{4}{5}$)

$\frac{2}{6}$ (or $\frac{1}{3}$)

Subtract each fraction from one whole.

$\frac{3}{5}$ $\frac{4}{7}$ $\frac{5}{12}$

$\frac{2}{9}$ $\frac{3}{4}$ $\frac{5}{8}$

What connections can you see between the fractions and your answers?

$\frac{2}{5}$, $\frac{3}{7}$, $\frac{7}{12}$

$\frac{7}{9}$, $\frac{1}{4}$, $\frac{3}{8}$

Two fractions have a difference of one half.

What could the fractions be?

Compare answers with a partner.

multiple possible answers, e.g.

$\frac{7}{4}$ and $\frac{5}{4}$

Subtract from a mixed number

Notes and guidance

In a previous step, children added to a mixed number as a prerequisite for adding mixed numbers; in this small step, they look at a similar process for subtracting. Children subtract either a whole number part or a fractional part from a mixed number. Crossing the whole is not included, as this is the focus of the next step.

Encourage children who need support to continue to use concrete resources and pictorial representations to make sense of the methods. This step provides more opportunities to develop their understanding of equivalent fractions, as some of the denominators are multiples of the other denominator in the calculation. Although some answers could be simplified, this is not the focus of the step.

Things to look out for

- Children may need support in partitioning mixed numbers.
- Children may overcomplicate the calculations by converting mixed numbers to improper fractions, which will not be necessary for subtractions of this type.
- Children may not pay attention to the calculation and add instead of subtract.

Key questions

- How can you partition a mixed number?
- Can the subtraction be written in a different form to make it easier?
- If the denominators are different, what do you need to do?
- How can the parts be combined to produce a mixed number?
- Do you need to combine whole numbers or fractions?
- Can you change the order of the numbers in a subtraction?

Possible sentence stems

- A mixed number can be partitioned into a ______ part and a ______ part.
- The difference between the wholes is ______
- The difference between the fractions is ______

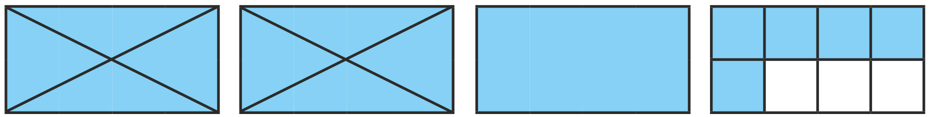
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

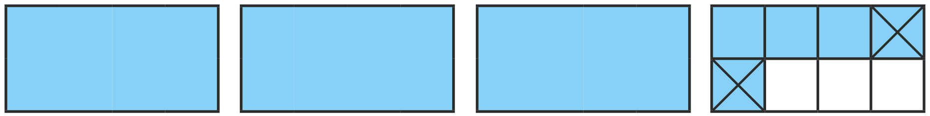
Subtract from a mixed number

Key learning

- Explain how the diagram shows $3\frac{5}{8} - 2 = 1\frac{5}{8}$



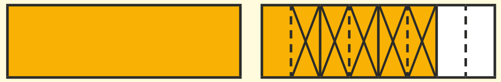
What calculation does this diagram show?



- Work out the subtractions.
  - $3\frac{4}{5} - 1$
  - $3\frac{4}{5} - 2$
  - $6\frac{3}{5} - 4$
  - $3\frac{4}{5} - \frac{1}{5}$
  - $3\frac{4}{5} - \frac{2}{5}$
  - $3\frac{4}{5} - \frac{3}{5}$

- Work out the missing numbers.
  - $4\frac{3}{4} -$ ______ $= 1\frac{3}{4}$
  - $3\frac{5}{7} -$ ______ $= 3\frac{1}{7}$
  - ______ $- \frac{2}{9} = 6\frac{5}{9}$

- Here is Ron's method for working out $1\frac{3}{4} - \frac{5}{8}$



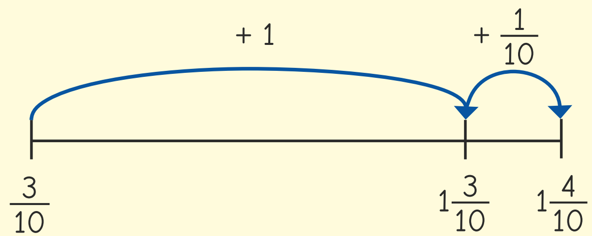
$1\frac{3}{4} - \frac{5}{8} = 1\frac{6}{8} - \frac{5}{8} = 1\frac{1}{8}$

Explain the steps in Ron's method.

Use Ron's method to work out the subtractions.

  - $2\frac{3}{5} - \frac{3}{10}$
  - $2\frac{2}{3} - \frac{1}{6}$
  - $1\frac{11}{12} - \frac{5}{6}$

- Kim uses a number line to find the difference between $1\frac{2}{5}$ and $\frac{3}{10}$



$1\frac{2}{5} = 1\frac{4}{10}$

difference $= 1 + \frac{1}{10} = 1\frac{1}{10}$

Find the difference between each pair of fractions.

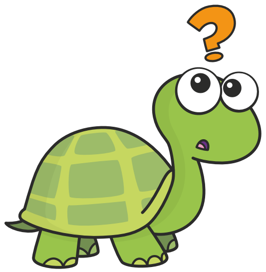
  - $3\frac{5}{6}$ and $\frac{1}{12}$
  - $\frac{11}{18}$ and $2\frac{7}{9}$
  - $5\frac{5}{7}$ and $\frac{3}{14}$

Subtract from a mixed number

Reasoning and problem solving

Tiny is trying to work out this subtraction.

$$2\frac{5}{14} - \frac{2}{7}$$



$$2\frac{5}{14} - \frac{2}{7} = 2\frac{3}{7}$$

Do you agree with Tiny?

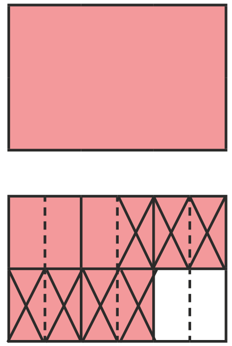
Explain your answer.

Work out the correct answer.

No

$2\frac{1}{14}$

Dora uses a diagram to work out a calculation.



What calculation is Dora working out?

What is the answer to the calculation?

$1\frac{5}{6} - \frac{7}{12}$ or $1\frac{10}{12} - \frac{7}{12}$

$1\frac{3}{12}$ or $1\frac{1}{4}$

Work out the subtraction.

$$5\frac{4}{9} - 3 - \frac{1}{3}$$

$2\frac{1}{9}$

Subtract from a mixed number – breaking the whole

Notes and guidance

There are many ways to subtract a fraction from a mixed number crossing the whole, and this small step encourages children to think flexibly about how to approach problems of this kind.

In addition to the methods illustrated in the Key learning section, children could also count back from the given fraction, providing the denominators are equal. This could be supported by the use of a number line. As in previous steps, either the denominators are equal, or one denominator is a multiple of the other.

Flexible partitioning and fluency in converting between improper fractions and mixed numbers are vital as children move from the pictorial to more abstract methods of recording their answers.

Things to look out for

- Children may not realise the need to cross the whole and instead subtract the smaller fraction part from the greater, for example working out $3\frac{1}{5} - \frac{4}{5}$ as $3\frac{4}{5} - \frac{1}{5}$ and giving the answer as $3\frac{3}{5}$
- Children may subtract numerators and denominators instead of finding a common denominator.
- Errors may occur with conversion between mixed numbers and improper fractions.

Key questions

- Which fraction is greater?
- How can you show the calculation as a diagram/on a number line?
- If the denominators are different, what do you need to do?
- How can you partition the mixed number? Is there more than one way?
- Is it easier to partition or to convert the mixed number to an improper fraction?
- Can you change the order of the numbers in a subtraction?

Possible sentence stems

- ______ can be partitioned into ______ and ______ or ______ and ______
- There are ______ in one whole, so there are ______ in ______

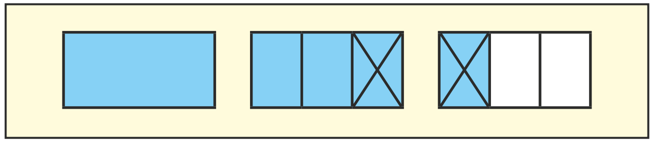
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Subtract from a mixed number – breaking the whole

Key learning

- Kim uses diagrams to show that $2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$



Work out the subtractions.

▸ $4\frac{1}{4} - \frac{3}{4}$ ▸ $3\frac{3}{8} - \frac{7}{8}$ ▸ $2\frac{2}{5} - \frac{4}{5}$

- Scott and Esther use improper fractions to show that $2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$

Scott

$2\frac{1}{3} - \frac{2}{3} = \frac{7}{3} - \frac{2}{3}$

$= \frac{5}{3}$

$= 1\frac{2}{3}$

Esther

$2\frac{1}{3} - \frac{2}{3} = 1\frac{4}{3} - \frac{2}{3}$

$= 1\frac{2}{3}$

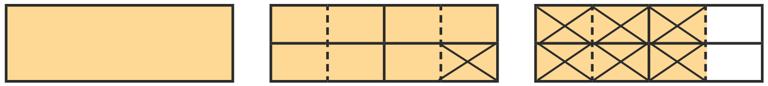
Explain why their methods work.

Whose method do you prefer?

Work out the subtractions.

▸ $3\frac{1}{5} - \frac{4}{5}$ ▸ $2\frac{2}{9} - \frac{7}{9}$ ▸ $3\frac{3}{7} - \frac{6}{7}$

- Here is a method for working out $2\frac{3}{4} - \frac{7}{8}$



Use this method to work out the subtractions.

▸ $3\frac{1}{3} - \frac{5}{6}$ ▸ $4\frac{1}{5} - \frac{7}{10}$ ▸ $5\frac{2}{3} - \frac{7}{9}$

- Tommy uses flexible partitioning to work out $6\frac{4}{9} - \frac{2}{3}$

$$6\frac{4}{9} - \frac{2}{3} = 5 + 1\frac{4}{9} - \frac{2}{3} = 5 + \frac{13}{9} - \frac{6}{9} = 5\frac{7}{9}$$

Use Tommy's method to work out the subtractions.

▸ $4\frac{2}{3} - \frac{5}{6}$ ▸ $3\frac{1}{5} - \frac{7}{15}$ ▸ $2\frac{1}{4} - \frac{7}{8}$

- Aisha has $3\frac{1}{4}$ bags of sugar.

She uses $\frac{7}{8}$ of a bag of sugar.

How much sugar does she have left?

Subtract from a mixed number – breaking the whole

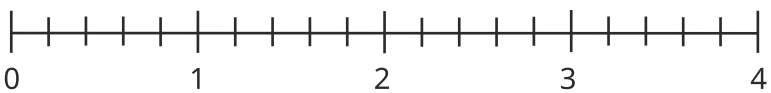
Reasoning and problem solving

Write the digits 2, 3 and 4 in the boxes to make the calculation correct.

$$27\frac{1}{\square} - \frac{\square}{6} = 26\frac{\square}{3}$$

$27\frac{1}{3} - \frac{4}{6} = 26\frac{2}{3}$

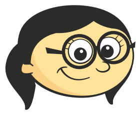
Show how you can use a number line to work out $2\frac{3}{5} - \frac{4}{5}$



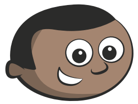
any correct method that illustrates $2\frac{3}{5} - \frac{4}{5} = 1\frac{4}{5}$

Three children are working out $6\frac{2}{3} - \frac{5}{6}$

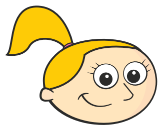
They all use partitioning to help them.



Annie: $5 + 1\frac{2}{3} - \frac{5}{6}$



Mo: $5 + 1\frac{4}{6} - \frac{5}{6}$



Eva: $5 + \frac{10}{6} - \frac{5}{6}$

Does each child have a correct starting point?

Explain your answer.

Yes

Subtract two mixed numbers

Notes and guidance

In this final small step of the block, children learn to subtract one mixed number from another.

Children begin by looking at simple cases where they partition two mixed numbers, then subtract the wholes and subtract the fractional parts. They then progress to more complex problems where they need to find a common denominator and/or break the whole.

As with earlier steps, there are a variety of possible approaches and these are explored, supported by diagrams. Children need to consider the most efficient approach for a given calculation rather than leaping into a method that might not be appropriate.

Things to look out for

- Children may assume they always need to convert to improper fractions, resulting in difficult arithmetic, when this is not necessary.
- Children may make errors when converting between mixed numbers and improper fractions.
- Children may subtract the denominators as well as the numerators.
- Children may subtract the fractions in the wrong order when they need to break the whole.

Key questions

- Is it possible to subtract the whole parts and fractional parts separately? Why or why not?
- Will you need to "break the whole"? Why or why not?
- Does making the whole numbers greater make the calculation more difficult? Why or why not?
- Is it easier to partition or to change the mixed number to an improper fraction?
- What diagrams could you use to support you?

Possible sentence stems

- The mixed numbers can be partitioned into a ______ part and a ______ part.
- When breaking the whole, the first number can be partitioned into ______ and ______

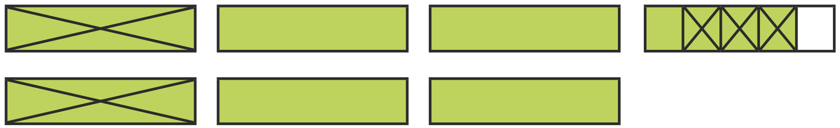
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Subtract two mixed numbers

Key learning

- The diagram illustrates $6\frac{4}{5} - 2\frac{3}{5}$

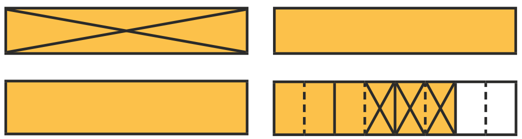


$6\frac{4}{5} - 2\frac{3}{5} = 4\frac{1}{5}$

Work out the subtractions.

$3\frac{5}{9} - 1\frac{1}{9}$ $\quad$ $6\frac{5}{7} - 4\frac{3}{7}$ $\quad$ $4\frac{13}{15} - 1\frac{2}{15}$

- Here is a bar model to help work out $3\frac{3}{4} - 1\frac{3}{8}$

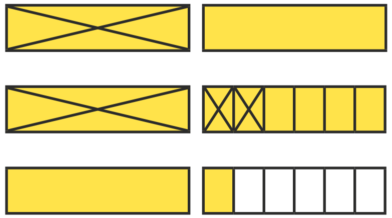


$3\frac{3}{4} - 1\frac{3}{8} = 2\frac{3}{8}$

Work out the subtractions.

$3\frac{7}{8} - 2\frac{3}{4}$ $\quad$ $5\frac{5}{6} - 2\frac{1}{3}$ $\quad$ $3\frac{2}{3} - 1\frac{5}{9}$

- Here is a method for working out $5\frac{1}{6} - 2\frac{1}{3}$



$5\frac{1}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{2}{6} = 2\frac{5}{6}$

Use this method to work out the subtractions.

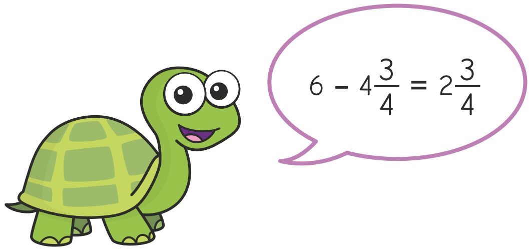
$3\frac{1}{4} - 2\frac{5}{8}$ $\quad$ $5\frac{1}{3} - 2\frac{7}{12}$ $\quad$ $27\frac{1}{3} - 14\frac{7}{15}$

- Compare the two methods for working out $5\frac{4}{15} - 1\frac{8}{15}$

Method 1	Method 2
$5\frac{4}{15} - 1\frac{8}{15} = 4\frac{19}{15} - 1\frac{8}{15}$ $= 3\frac{11}{15}$	$5\frac{4}{15} - 1\frac{8}{15} = \frac{79}{15} - \frac{23}{15}$ $= \frac{56}{15}$ $= 3\frac{11}{15}$

Subtract two mixed numbers

Reasoning and problem solving



Explain why Tiny is wrong.

Find the correct answer.

$1\frac{1}{4}$

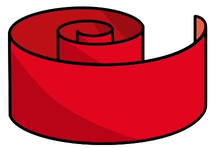
A bag of dog food contains only red, brown and orange biscuits.

- The total mass of the dog food is 7 kg.
- The mass of red biscuits is $3\frac{3}{4}$ kg.
- The mass of brown biscuits is $1\frac{7}{16}$ kg.

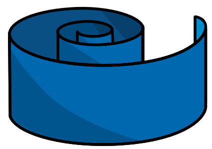
What is the mass of the orange biscuits?

$1\frac{13}{16}$ kg

Rosie has $20\frac{3}{4}$ cm of ribbon.



Annie has $6\frac{7}{8}$ cm less ribbon than Rosie has.



How much ribbon does Annie have?

How much ribbon do Annie and Rosie have altogether?

Compare methods with a partner.

$13\frac{7}{8}$ cm

$34\frac{5}{8}$ cm